ICS6B Assignment 4

Due: Thursday, 10th May, 2018 by 7am on Gradescope

1 Sets

1. Membership Consider the set $S = \{ \text{integers divisible by 3} \}$.
   (a) Show that if $a \in S$ and $b \in S$, then $ab \in S$.
   (b) Show that if $a \in S$, then $\exists b \in S$, such that $a + b = 0$.
   (c) Suppose $a \in S$. Under what conditions would $\frac{a}{2} \in S$?

2. Set Relationships and Equivalences Let $A, B, C$ be sets. Show the following; Venn diagrams may be used as long as they cover all relevant cases.
   (a) If $A \cap C = B \cap C$, does $A = B$?
   (b) If $A \setminus B = A$, what does this mean about $B$?
   (c) Show that $(B \setminus A) \cup (C \setminus A) = (B \cup C) \setminus A$
   (d) Show that $(A \setminus C) \cap (C \setminus B) = \emptyset$

2 Functions

1. Injections and Surjections Consider the set of integers from 1 to 10, $P = 0, 1, 2, 3, ..., 10$.
   (a) Show that the function $f : P \mapsto \mathbb{Z}$ defined as $f(x) = 2x$ is injective but not surjective. Is it a bijection?
   (b) Show that the function $f : [0, 100] \mapsto P$ defined as $f(x) = \lfloor \frac{x}{10} \rfloor$ is surjective but not injective. Note that $\lfloor x \rfloor$ is the floor function. $\lfloor x \rfloor$ is the largest integer less than or equal to $x$. Is $f$ a bijection?

2. Bijections (1 to 1)
   (a) Show that there is a bijection between the set of even integers and the set of odd integers.
   (b) Show that the function $f(x) = \frac{2x-1}{2^x(x-1)}$ is a bijection from $(0, 1)$ to $\mathbb{R}$

3. Funny Function Define a function $f$ on the positive integers, $f : \mathbb{Z}_+ \mapsto \mathbb{Z}_+$ such that $f(n+1) > f(n)$ and $f(f(n)) = 3n$. Find the value of $f(10)$.