ICS6B Assignment 5

Due: Thursday, 17th May, 2018 by 7am on Gradescope

1 Relations

1. Let $S$ be a set and $R_1$ and $R_2$ are relations on $S$. Prove the following if they are true. If it is false, give a counter-example.

   (a) If $R_1$ and $R_2$ are reflexive, is $R_1 \cup R_2$ reflexive?

   (b) If $R_1$ is reflexive, is $R_1^{-1}$ reflexive?

   (c) If $R_1$ and $R_2$ are symmetric, is $R_1 \circ R_2$ symmetric?

   (d) If $R_1$ and $R_2$ are anti-symmetric, is $R_1 \oplus R_2$ anti-symmetric?

   (e) If $R_1$ is anti-symmetric, is $R_1^n$ anti-symmetric, $n \in \mathbb{Z}$?

2. Consider the set of integers $\mathbb{Z}$ with relation $R$ where $xRy$ means $y$ is the remainder when 7 divides $x^2$.

   (a) Write the pairs $(x, y)$ formed by relation $R$ for $x = 1, 2, 3, \ldots, 14$.

   (b) Show that the range of $y$ is $\{0, 1, 2, 4\}$ under this relation.

   (c) Find the range of $y$ if $xRy$ is redefined to be the remainder when 5 divides $x^2$.

   (d) Is $R$ symmetric?

   (e) Is $R$ reflexive?

   (f) Is $R$ transitive?

3. Consider a set $A$ with a relation $R$. Show that if $R$ is transitive and symmetric, then $R$ is reflexive.

4. Consider the ordered set of elements $S = \{a, b, c, f, d\}$ with the relation $R$ where $xRy$ means if element $x$ occupies position $i$ in $S$, then it is related to the element in position $i + 2$. The position loops around, so from position 5 we go back to 1.

   (a) Write the elements of $R$.

   (b) Is $R$ symmetric?

   (c) Is $R$ anti-symmetric?
(d) Is \( R \) reflexive?
(e) Is \( R \) transitive?
(f) Redo the above questions for the case where the position of elements do not loop.