ICS6B Assignment 6

Due: Tuesday, 29th May, 2018 by 7am on Gradescope

1 Transitivity

1. Let $S$ be the set $\{\text{monkey, horse, cow, piglet, lamb}\}$ and $R$ be a relation defined on $S$ containing the pairs: $(\text{monkey, cow}), (\text{horse, piglet}), (\text{cow, lamb}), (\text{cow, monkey}), (\text{lamb, horse}), (\text{lamb, piglet}), (\text{lamb, monkey})$

Find $R^*$ and show your steps.

2. Suppose $R$ is a relation on a set $S$. Prove the following:
   (a) If $R$ is symmetric, then $R^*$ is symmetric.
   (b) If $R$ is reflexive, then $R^*$ is reflexive.

3. Let $S$ be the set defined in Question 1. Use Warshall’s algorithm to find the transitive closures of the following relations on $S$:
   (a) $\{(\text{horse, cow}), (\text{horse, lamb}), (\text{lamb, cow}), (\text{lamb, horse}), (\text{piglet, monkey}), (\text{cow, lamb})\}$
   (b) $\{(\text{monkey, horse}), (\text{monkey, cow}), (\text{horse, cow}), (\text{monkey, lamb}), (\text{horse, monkey}), (\text{cow, horse}), (\text{piglet, monkey}), (\text{lamb, piglet}), (\text{cow, monkey})\}$

4. Consider the relation given in Question 1.
   (a) What is the zero-one matrix $M_R$ of the relation?
   (b) What is $M_R^3 \cdot M_R^4$? Show your work.
   (c) What is the zero-one matrix of the transitive closure of $R^*$? Show your work.