Fast Convergence in the Double Oral Auction[1]

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Introduction

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$\Rightarrow$ Prices did not tend to equilibrium prices!

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⇒ 2002 Smith received Nobel prize for his founding role in experimental economics

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How do these markets overcome private information and suboptimal behavior to converge to equilibrium prices quickly?

Formal Setup

Definition (Market)

A market consists of a bipartite graph $G = (B, S, E)$ on a set B of buyers and a set S of sellers together with private valuations v_i for all $i \in B \cup S$.

We assume that $|B| = |S| = n$.

Instead of considering multiple trading days for small quantities of items, we consider a contract negotiation setting.

Formal Setup II

Definition (State)

A state consists of a matching $M \subseteq G$ and an assignment of offers (bids / asks) $p : B \cup S \to \mathbb{R}_{\geqslant 0}$.

A state is considered valid iff

- for all $b \in B$, we have $p_b \leqslant v_b$,
- for all $s \in S$, we have $p_s \geqslant v_s$, and
- for all $(b, s) \in M$ we have $p_b \geqslant p_s$.

We will assume that the market is always in a valid state!

The Double Oral Auction

Definition (Interested Players)

A seller S is interested in his neighbor B if $p_b > p_s$ or if S is not matched and $p_b \geqslant p_s$; likewise for buyers.

A single iteration of the DOA consists of:

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- Among interested neighbors, unmatched neighbors are given priority.

Results

Definition (Social Welfare)

The social welfare of a valid state is defined as

$$\mathrm{SW} := \sum_{(b,s)\in M} (v_s - v_b).$$

We will show:

Theorem

If all prices in the DOA are multiples of $\epsilon > 0$ and the market graph is complete, then the mechanism converges in at most $O(n^3/\epsilon)$ iterations to a valid state which is within $n\epsilon$ of optimal social welfare.

Results II

Theorem

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Theorem

If all prices increases in the DOA* are by exactly $\epsilon > 0$ and active players are chosen uniformly at random, then with high probability, the mechanism converges in at most $O(n^3 \log n/\epsilon^2)$ iterations to a valid state which is within $n\epsilon$ of optimal social welfare.

Complete Bipartite Graphs

Stability

Definition

A state is called ϵ-stable for some $\epsilon > 0$, if

1. for all $(b, s) \in E$, we have $p_b \leqslant p_s + \epsilon$,
2. for all unmatched players i, we have $p_i = v_i$, and
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Using LP duality one can show fairly easily:

Lemma

If a valid state is ϵ-stable, then its social welfare is within $n\epsilon$ of the optimum social welfare.

Goal

Our goal is thus:

Theorem

If all prices in the DOA are multiples of $\epsilon > 0$ and the market graph is complete, then the mechanism converges in at most $O(n^3/\epsilon)$ iterations to a valid ϵ-stable state.

Main Lemmas

Lemma

Under the assumptions of the theorem, any state which is reached by the DOA is valid and satisfies that for any $(b, s) \in E$ we have $p_b \leqslant p_s + \epsilon$.

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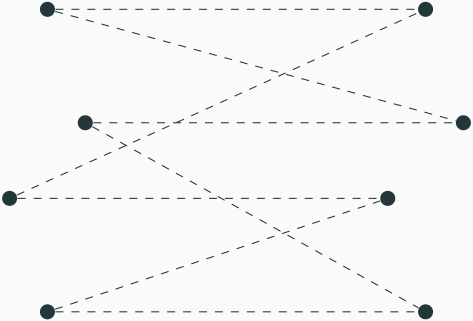
Under the assumptions of the theorem, any state which is reached by the DOA satisfies that for any $(b, s) \in E$, if $p_b > p_s$, then b and s are both matched.

General Bipartite Graphs

A Pathological Example

On incomplete graphs, the DOA may not always converge!



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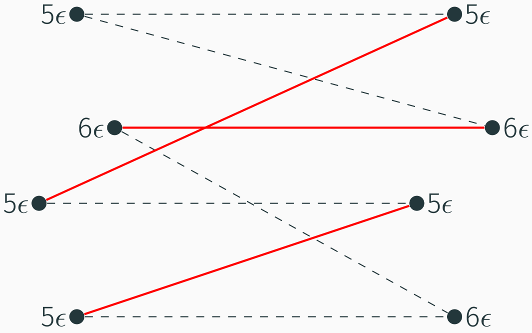

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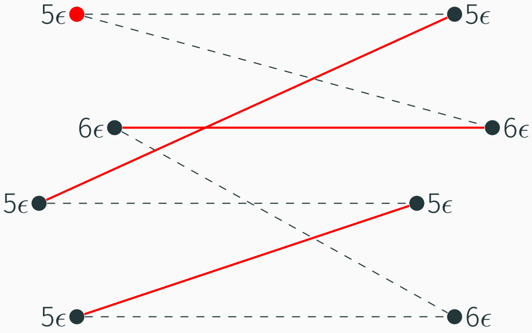

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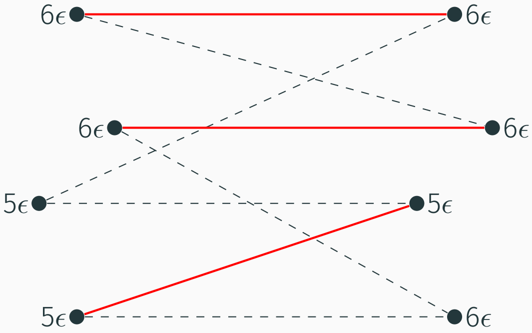

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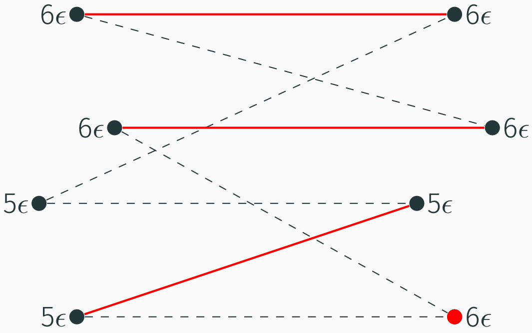

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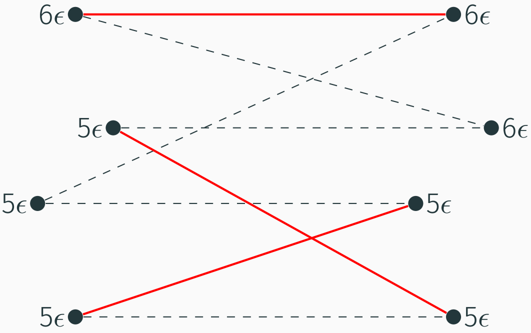

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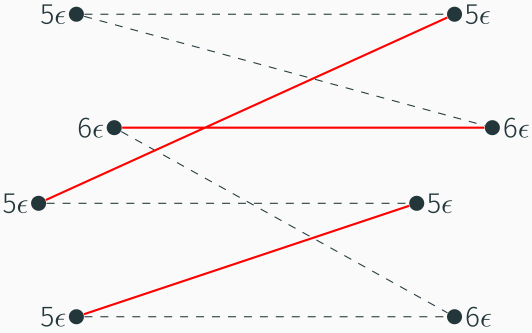

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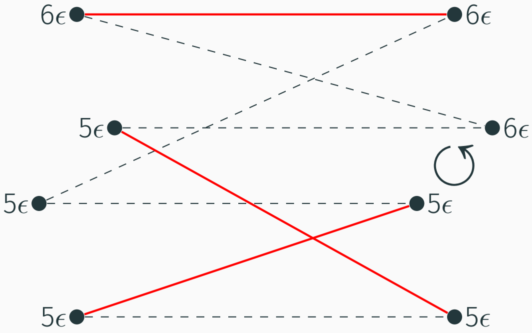

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As before: assume players don't overbid.

Main Theorem

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Thank You!