Student’s t-test

A t-test is any statistical hypothesis test in which the *test statistic* follows a Student’s t distribution if the null hypothesis is true.

Student's t-distribution

Suppose X_1, X_2, …, X_n are independent random variables that are normally distributed with mean μ and variance σ^2.

$$\text{sample mean: } \bar{X}_n = (X_1 + \ldots + X_n)/n$$

$$\text{sample variance: } S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$$

$$T = \frac{\bar{X}_n - \mu}{S_n/\sqrt{n}} \qquad Z = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}}$$

Z is normally distributed with mean 0 and variance 1
T has a Student's *t*-distribution with *n-1 degrees of freedom*.

The t-distribution looks like the standard normal distribution (exact with $n \to +\infty$) with fatter tails.

Independent one-sample t-test

Independent one-sample t-test:

Null hypothesis: the population mean is equal to a specified value μ_0.

Test static:

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

s: the sample standard deviation
n: sample size
Degree of freedom (d.o.f) = n-1

Independent two-sample t-test

1): Equal sample size, equal variance

Null hypothesis: the population mean is equal.

Test static:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_{X_1X_2}\sqrt{\frac{2}{n}}}$$

$$\text{where } S_{X_1X_2} = \sqrt{\frac{S_{X_1}{}^2 + S_{X_2}{}^2}{2}}$$

- S_{X1},S_{X2}: the sample standard deviation from each group.
- *n*: participants of *each* group
- Degree of freedom (d.o.f) = 2n-2

Independent two-sample t-test

2): Unequal sample size, equal variance

Null hypothesis: the population mean is equal.

Test static:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_{X_1X_2}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\text{where } S_{X_1X_2} = \sqrt{\frac{(n_1 - 1)S_{X_1}{}^2 + (n_2 - 2)S_{X_2}{}^2}{n_1 + n_2 - 2}}$$

- S_{X1},S_{X2}: the sample standard deviation from each group.
- *n*: participants of *each* group
- Degree of freedom (d.o.f) = 2n-2