Complexity; Nearest Neighbors

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CS 273P Machine Learning and Data Mining

Slides courtesy of Alex Ihler

Machine Learning

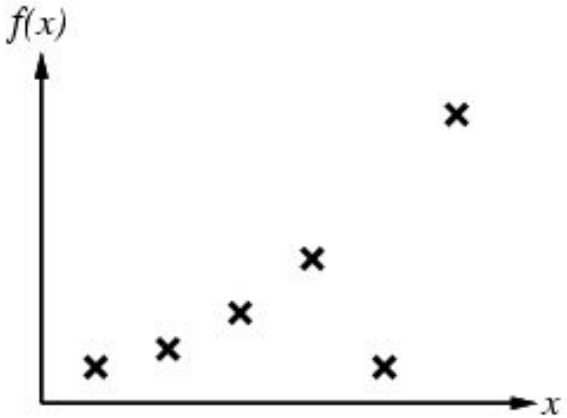
Complexity and Overfitting

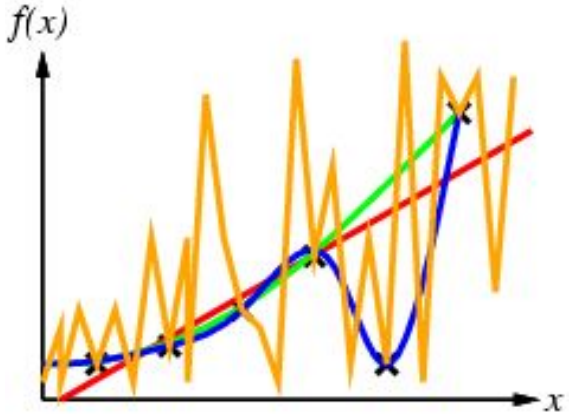
Nearest Neighbors

K-Nearest Neighbors

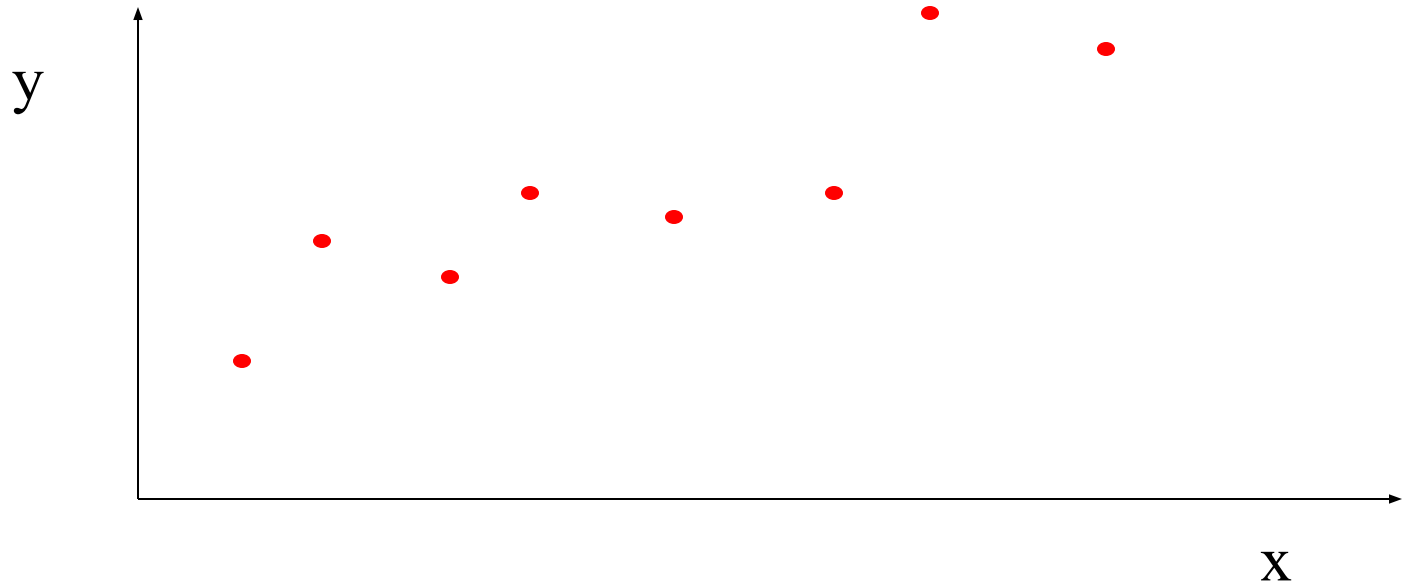
Inductive bias

- "Extend" observed data to unobserved examples
  - "Interpolate" / "extrapolate"
- What kinds of functions to expect? Prefer these ("bias")
  - Usually, let data pull us away from assumptions only with evidence!



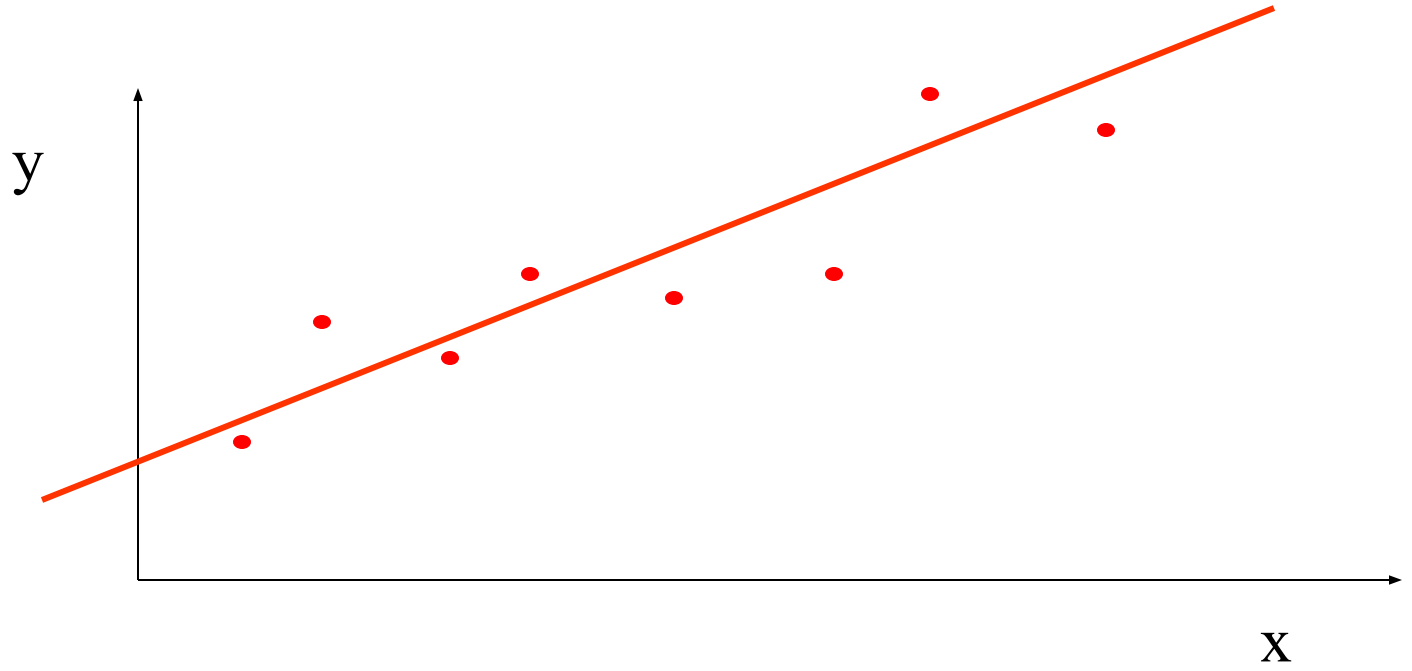


Overfitting and complexity

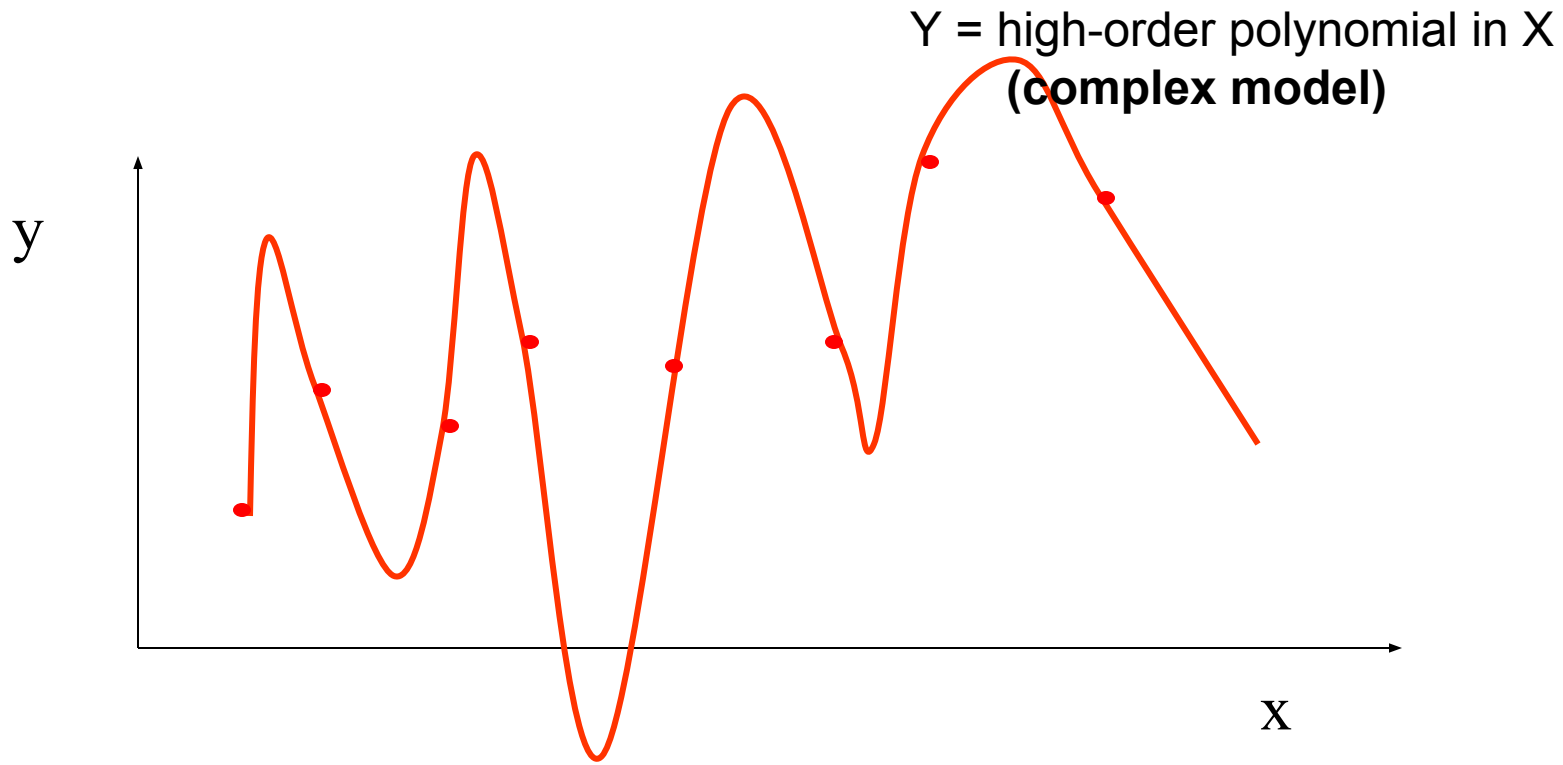


Overfitting and complexity

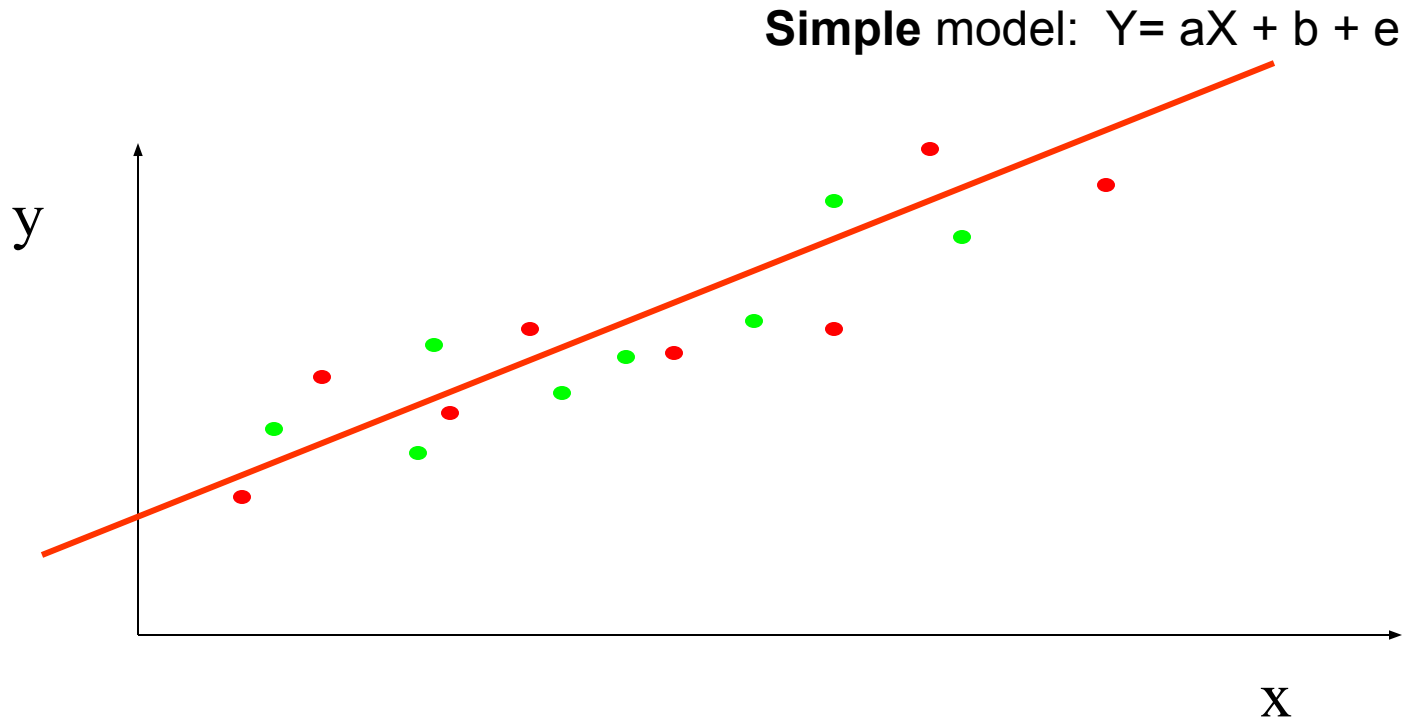
Simple model: Y= aX + b + e



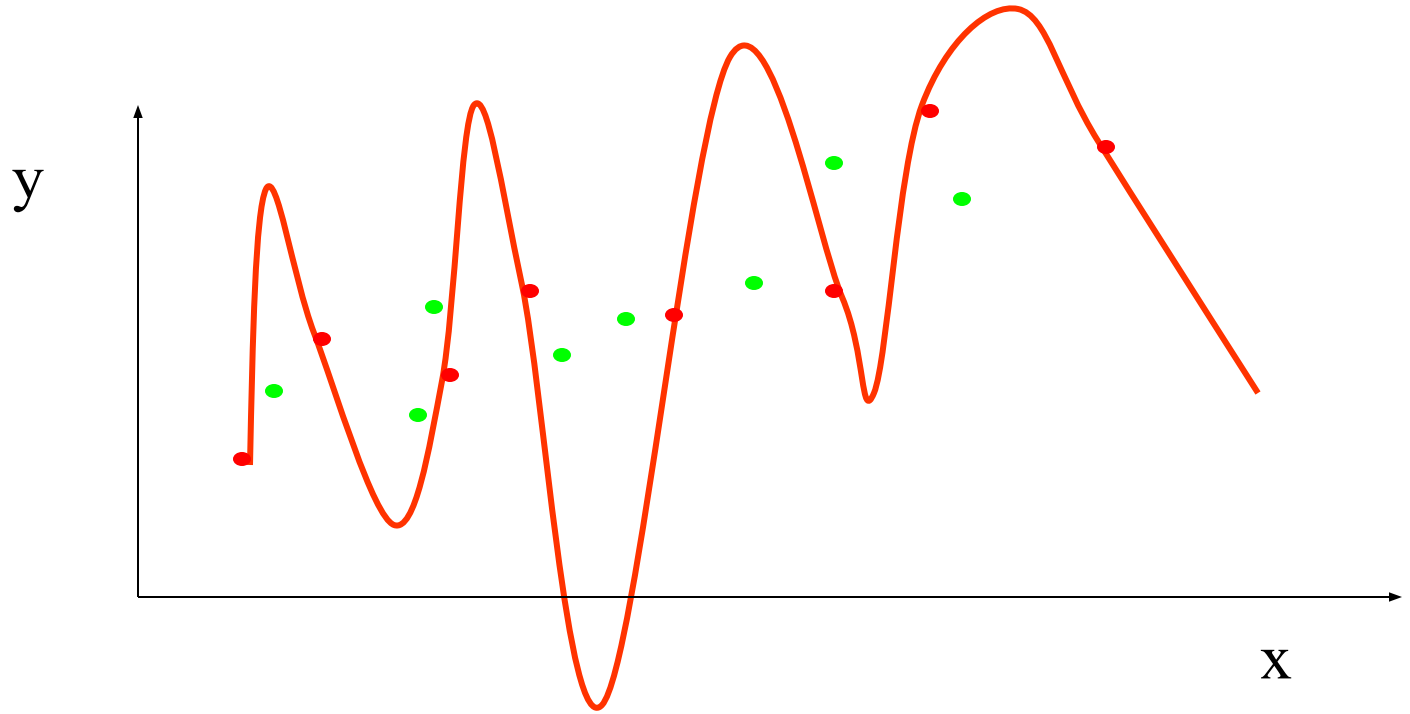
Overfitting and complexity



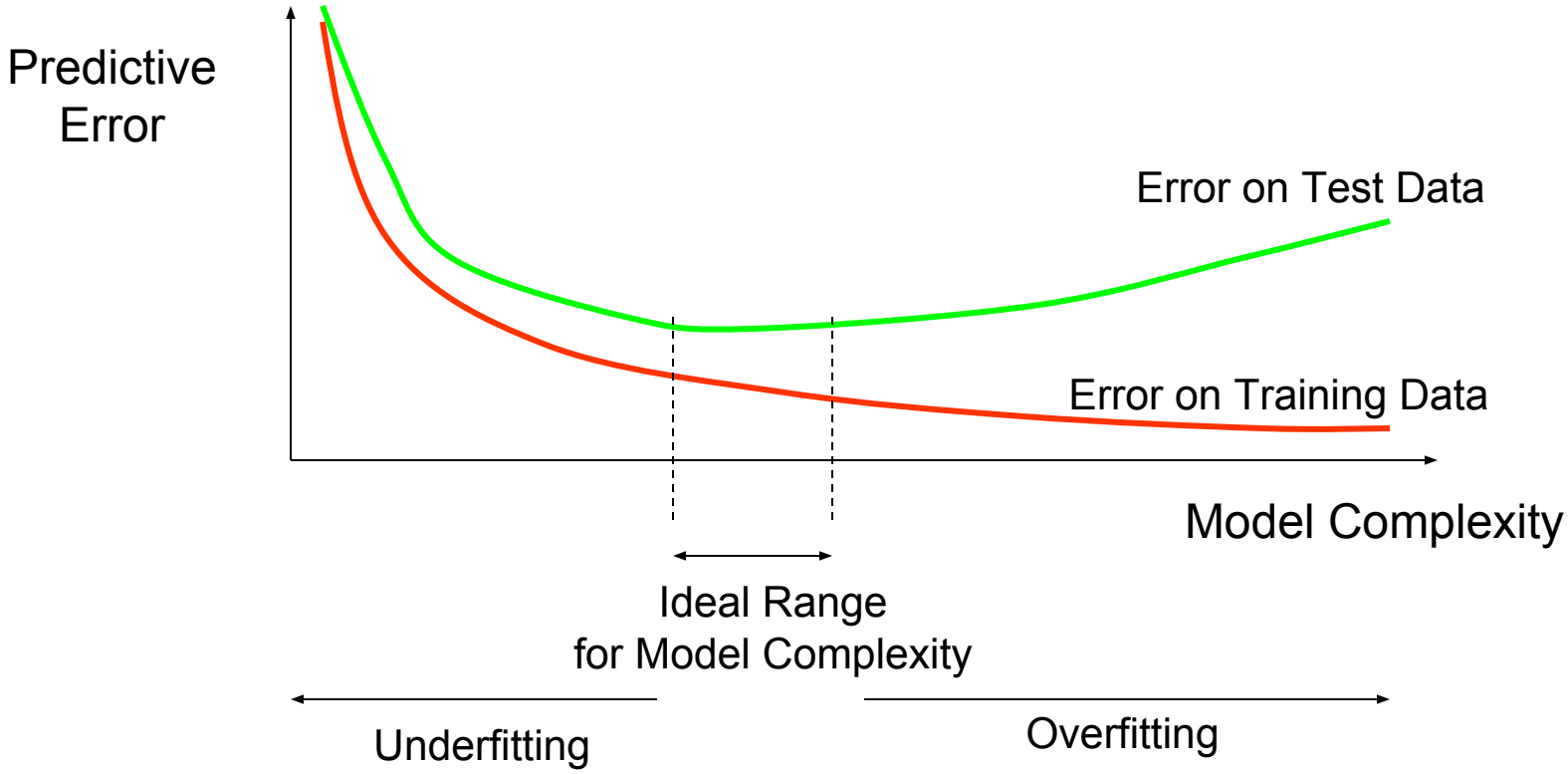
Overfitting and complexity



Overfitting and complexity



How Overfitting affects Prediction



Training and Test Data

- Several candidate learning algorithms or models, each of which can be fit to data and used for prediction
- How can we decide which is best?

Approach 1: Split into train and test data

- Learn parameters of each model from training data
- Evaluate all models on test data, and pick best performer

Problem:

- Over-estimates test performance (“lucky” model)
- Learning algorithms should *never* have access to test data

Training, Validation, and Test Data

- Several candidate learning algorithms or models, each of which can be fit to data and used for prediction
- How can we decide which is best?

Approach 2: Reserve some data for validation

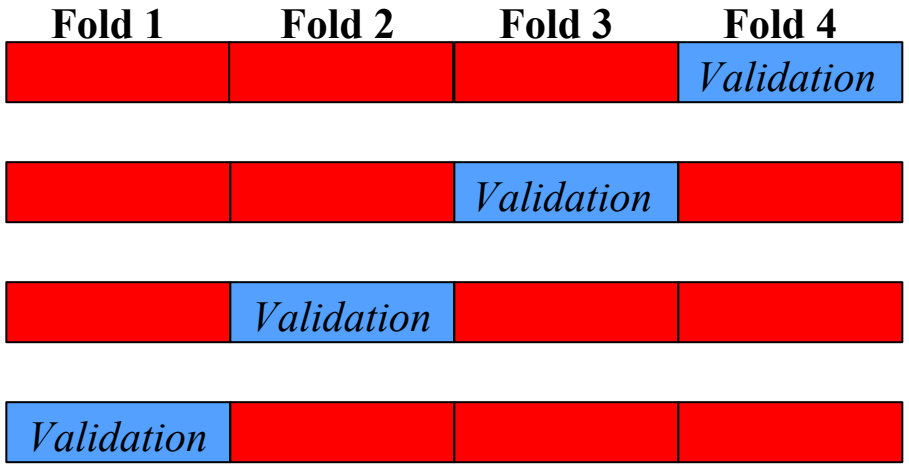
- Learn parameters of each model from training data
- Evaluate models on validation data, pick best performer
- Reserve test data to benchmark chosen model

Problem:

- Wasteful of training data (learning can't use validation)
- May bias selection towards overly simple models

Cross-Validation

- Divide training data into K equal-sized *folds*
- Train on K-1 folds, evaluate on remainder
- Pick model with best average performance across K trials



How many folds?

- *Bias:* Too few, and effective training dataset much smaller
- *Variance:* Too many, and test performance estimates noisy
- *Cost:* Must run training algorithm once per fold (parallelizable)
- *Practical rule of thumb:* 5-fold or 10-fold cross-validation
- *Theoretically troubled:* Leave-one-out cross-validation, K=N

Competitions

- Training data
  - Used to build your model(s)
- Validation data
  - Used to assess, select among, or combine models
  - Personal validation; leaderboard; …
- Test data
  - Used to estimate “real world” performance

#	Δ1w	Team Name * in the money	Score	Entries	Last Submission UT
1	-	BrickMover *	1.21251	40	Sat, 31 Aug 2013 23:
2	new	vsu *	1.21552	13	Sat, 31 Aug 2013 20:
3	↑2	Merlion	1.22724	29	Sat, 31 Aug 2013 23:
4	↓2	Sergey	1.22856	15	Sat, 31 Aug 2013 23:
5	new	liuyongqi	1.22980	13	Sat, 31 Aug 2013 13:

Machine Learning

Complexity and Overfitting

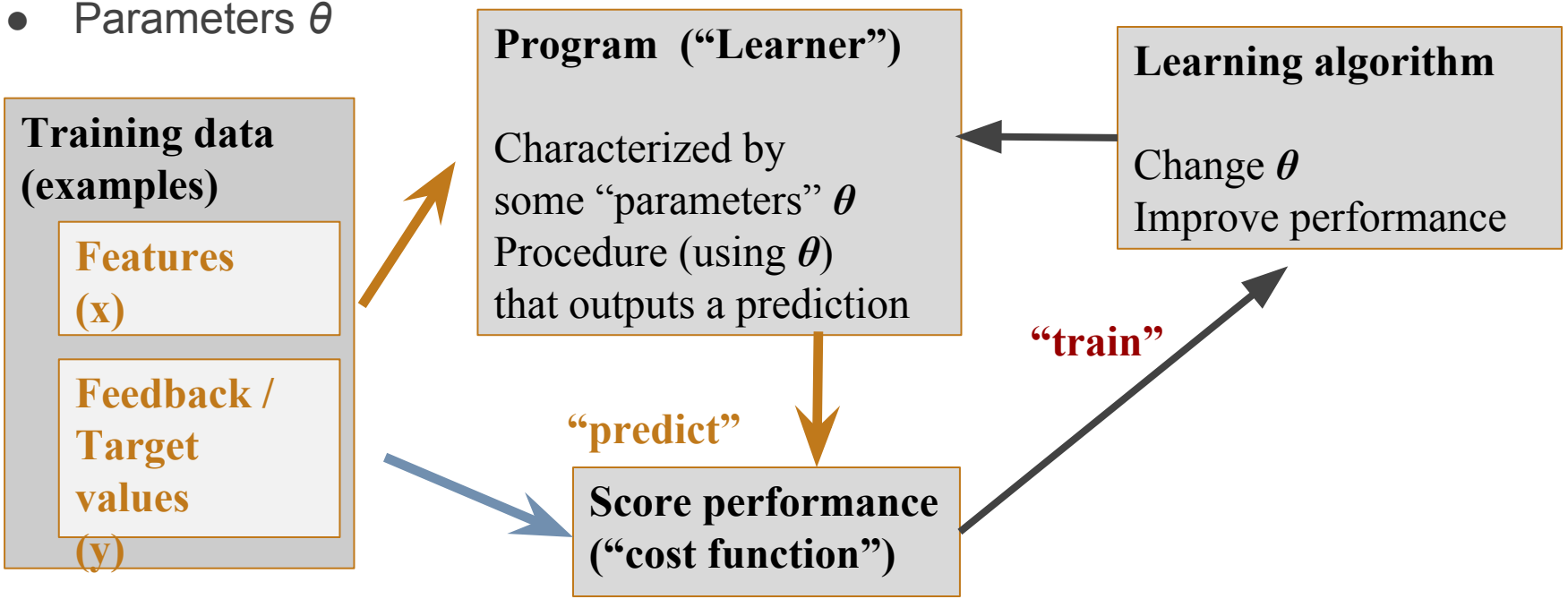
Nearest Neighbors

K-Nearest Neighbors

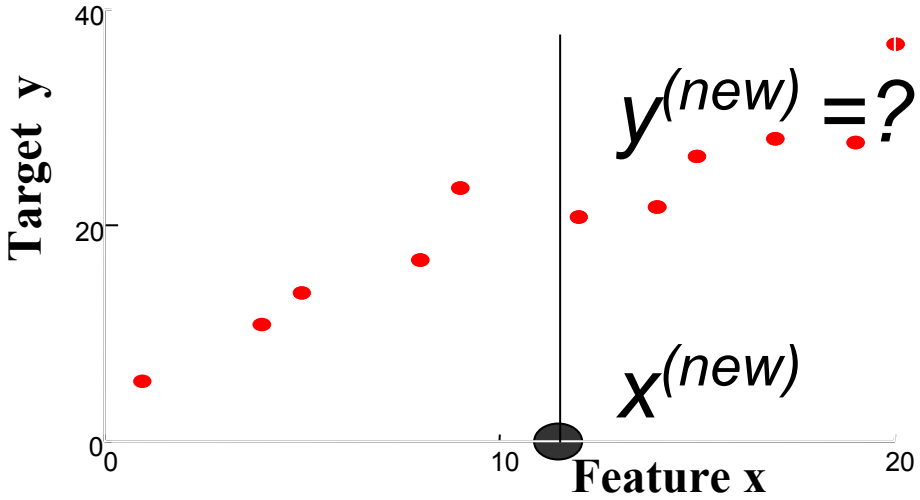
How does machine learning work?

Notation

- Features x
- Targets y
- Predictions $\hat{y} = f(x\ ;\ \theta)$
- Parameters θ

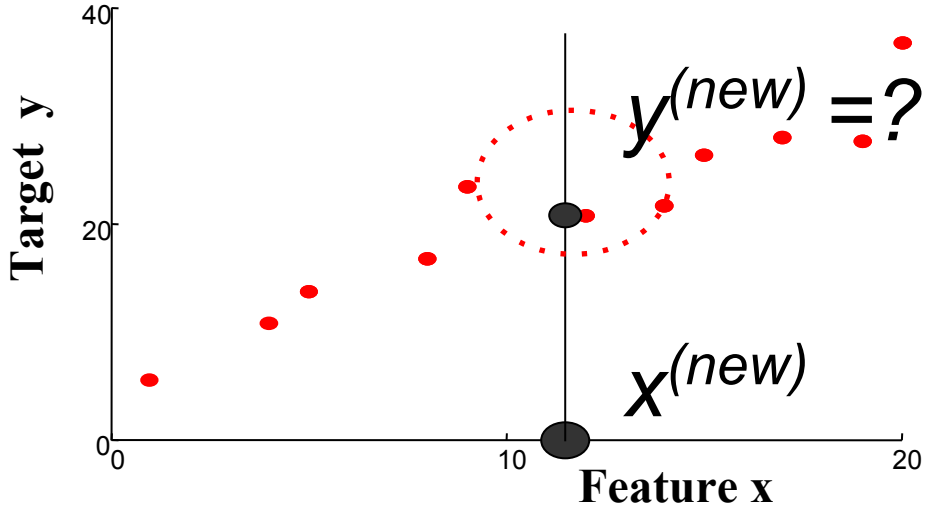


Regression; Scatter plots



- Suggests a relationship between x and y
- Regression: given new observed $x^{(new)}$, estimate $y^{(new)}$

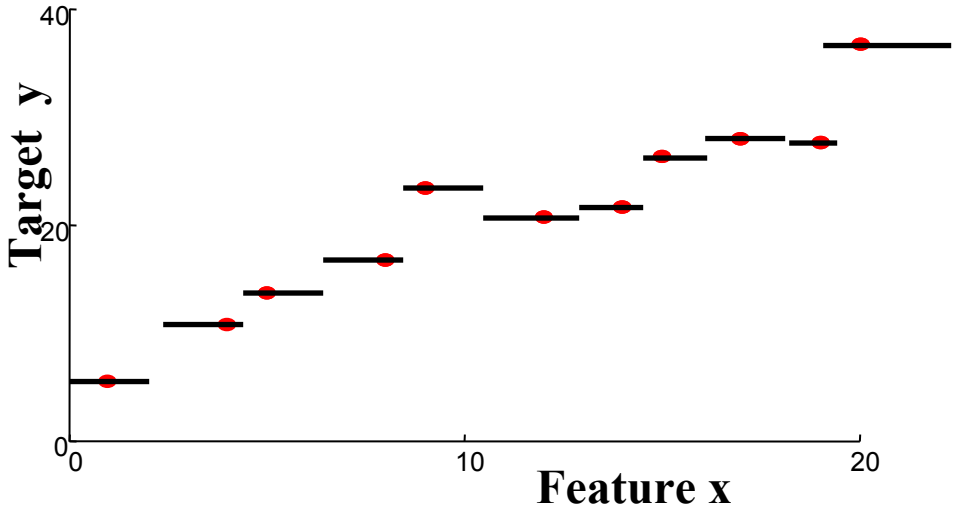
Nearest neighbor regression



"Predictor":
Given new features:
Find nearest example
Return its value

Find training datum $x^{(i)}$ closest to $x^{(new)}$; predict $y^{(i)}$

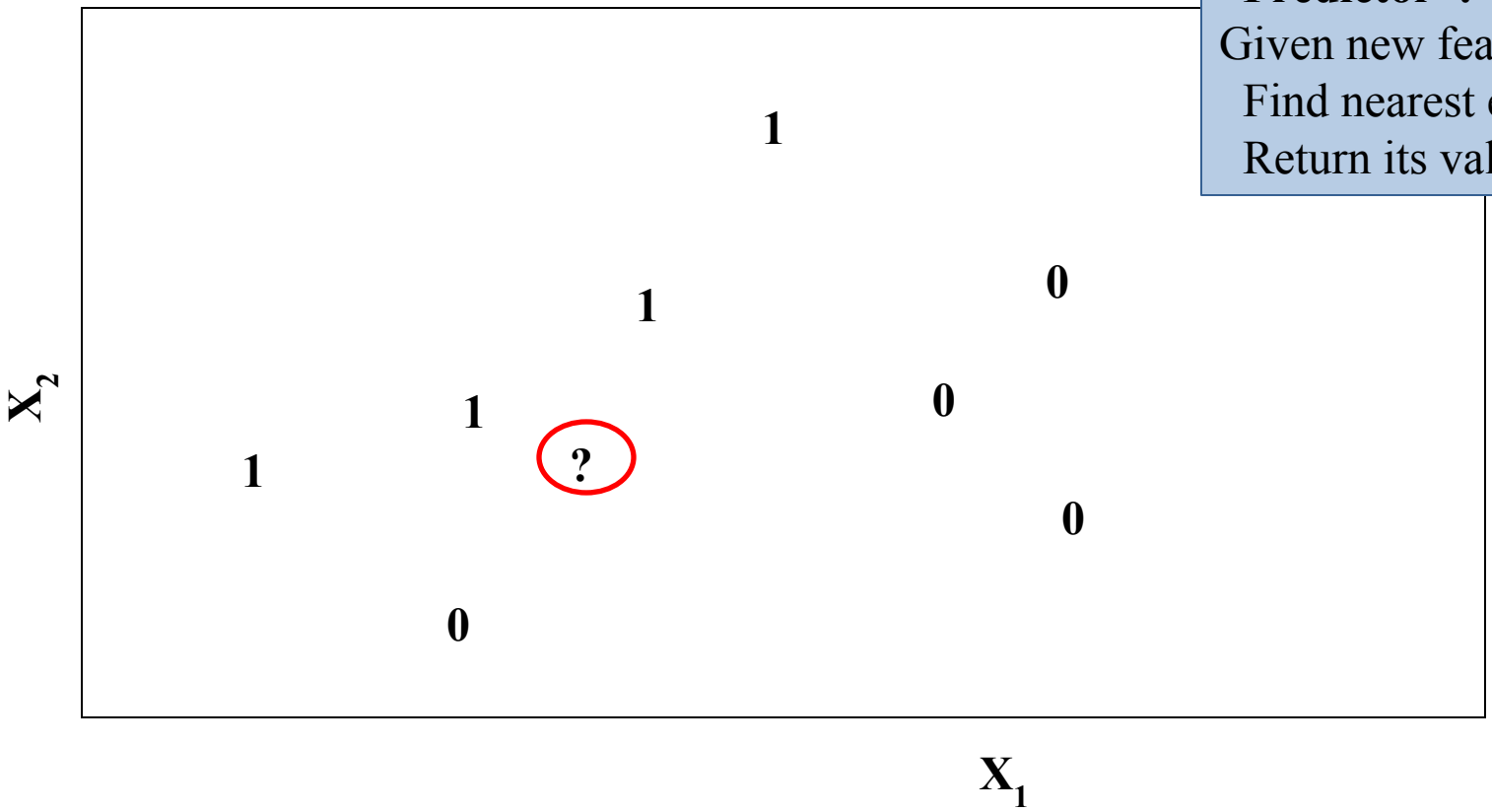
Nearest neighbor regression



"Predictor":
Given new features:
Find nearest example
Return its value

- Find training datum $x^{(i)}$ closest to $x^{(new)}$; predict $y^{(i)}$
- Defines an (implicit) function f(x)
- "Form" is piecewise constant

Nearest neighbor classifier



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Given new features:
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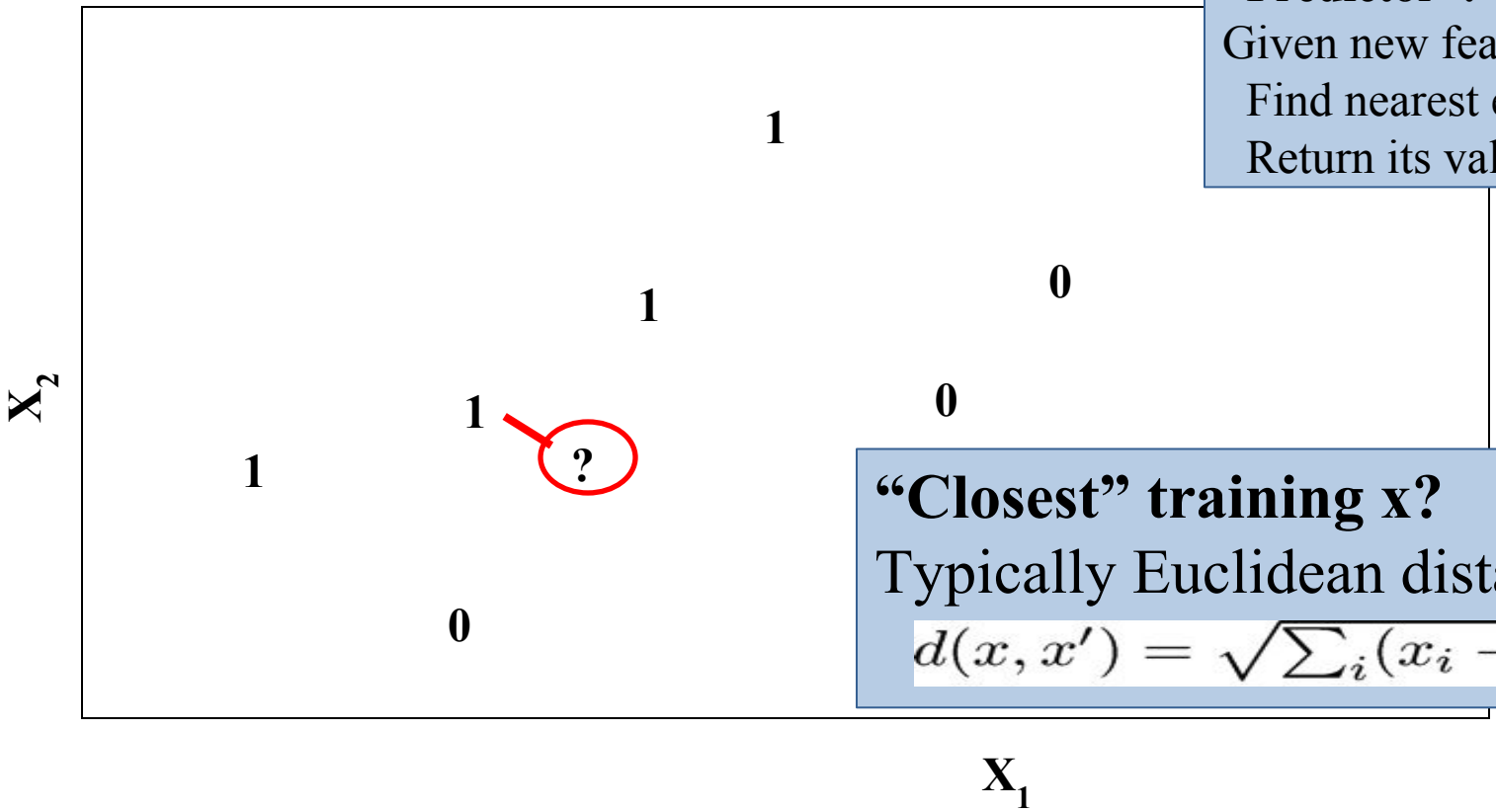
Nearest neighbor classifier

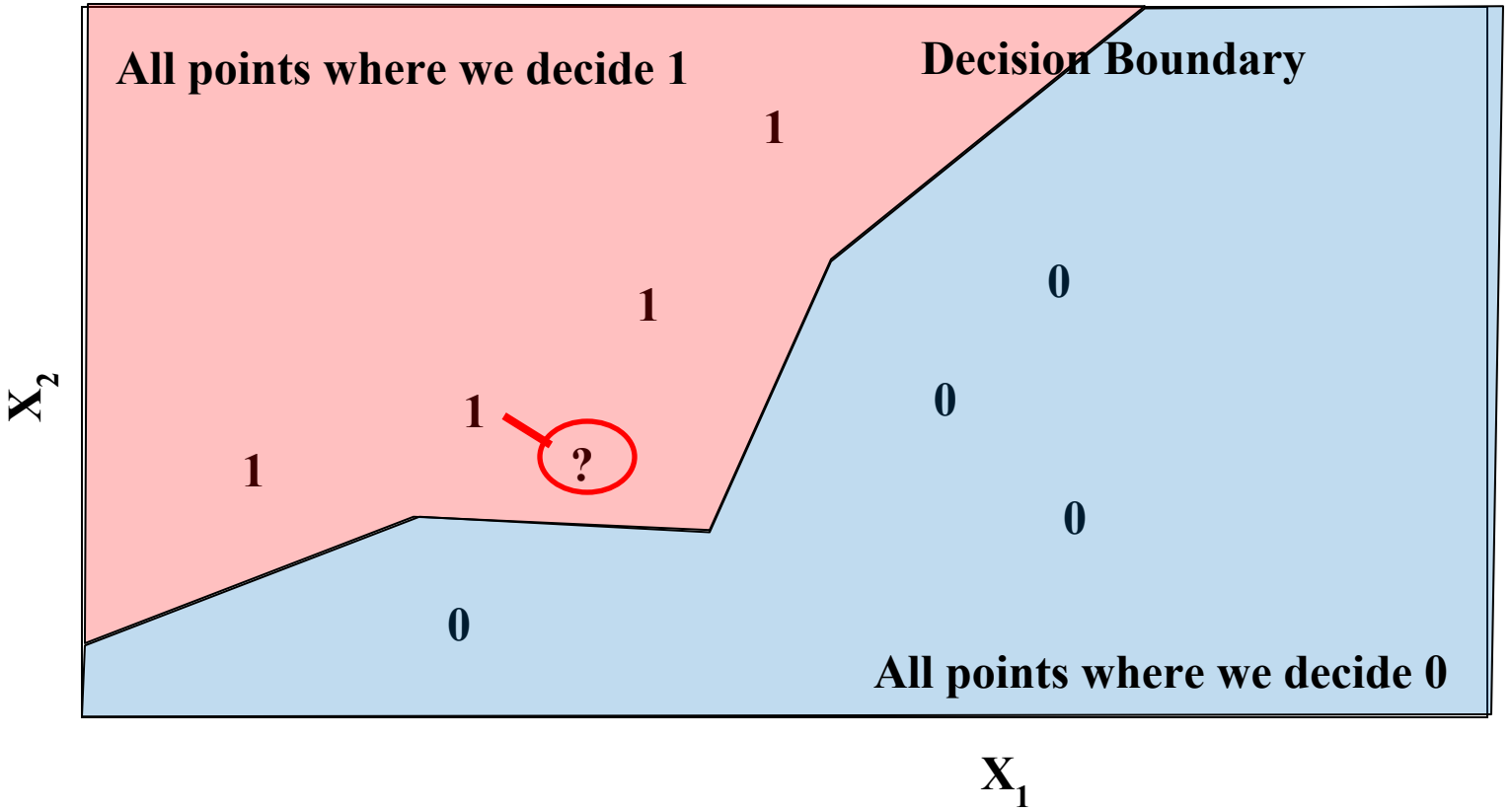


"Predictor":
Given new features:
Find nearest example
Return its value

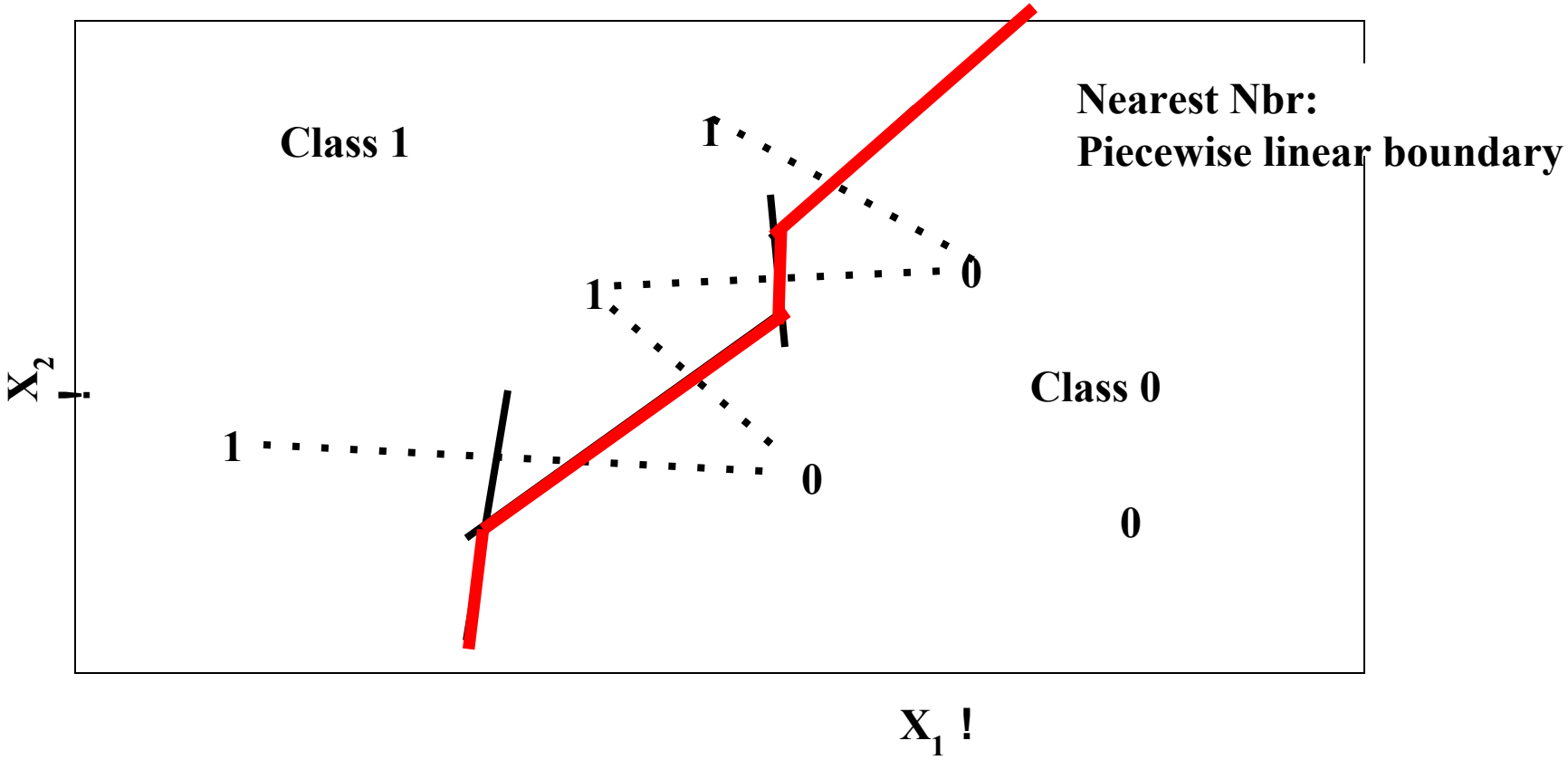
"Closest" training x?
Typically Euclidean distance:

$$d(x, x') = \sqrt{\sum_i (x_i - x'_i)^2}$$

Nearest neighbor classifier

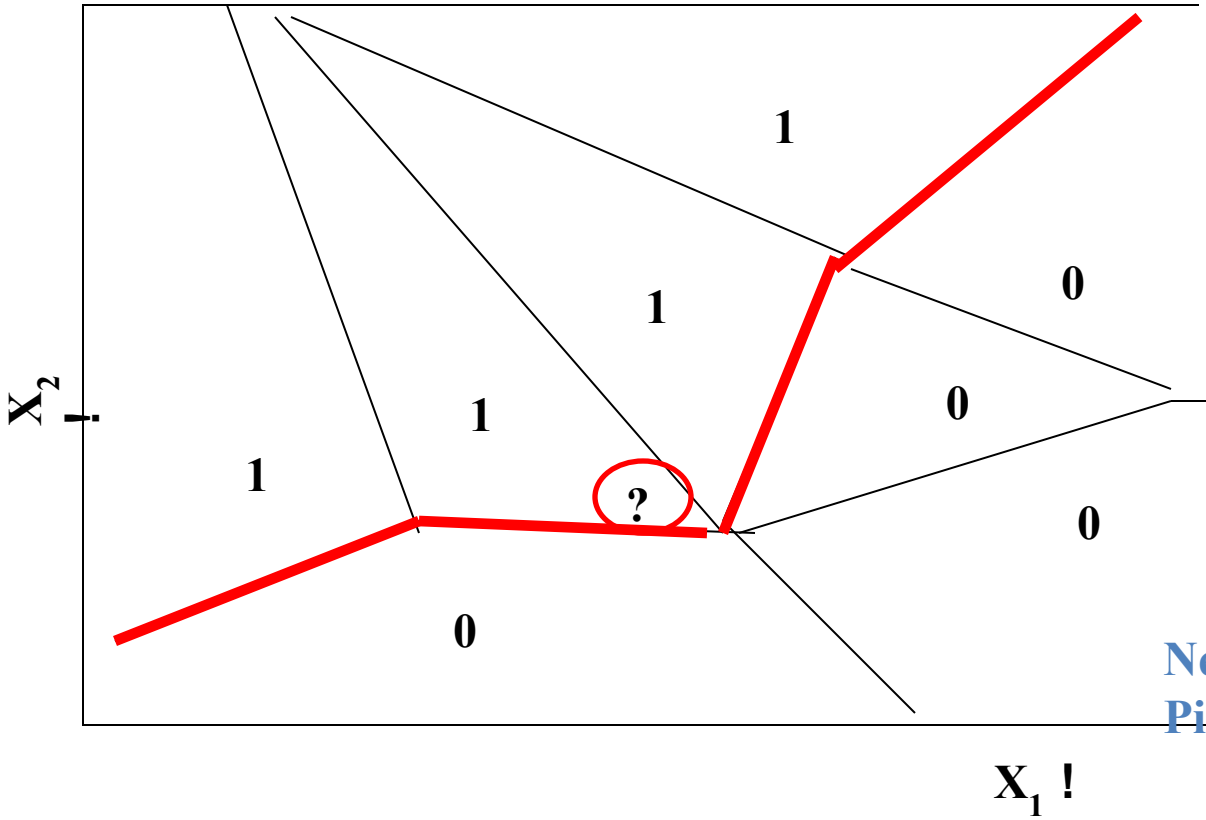


Nearest neighbor classifier



Voronoi Diagram

Nearest neighbor classifier

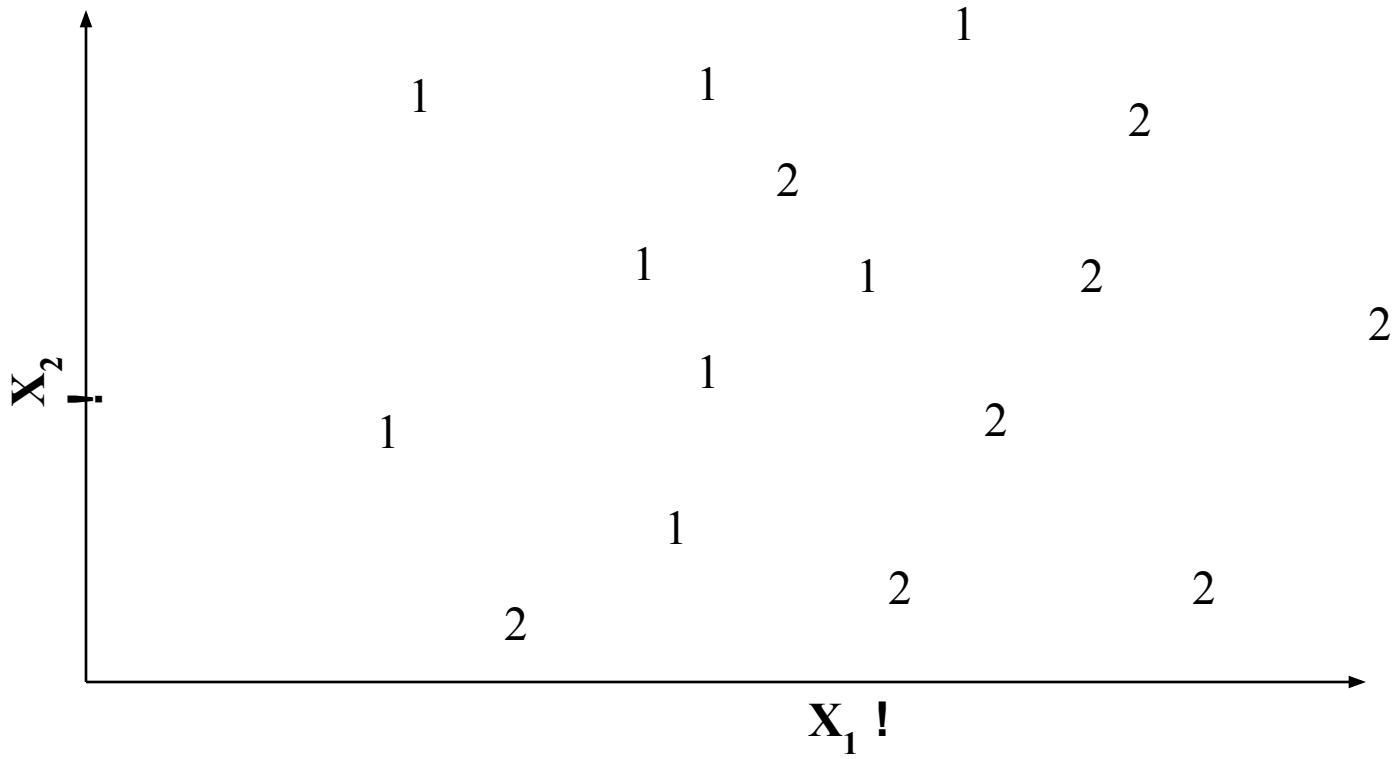


Voronoi diagram: Each datum is assigned to a region, in which all points are closer to it than any other datum

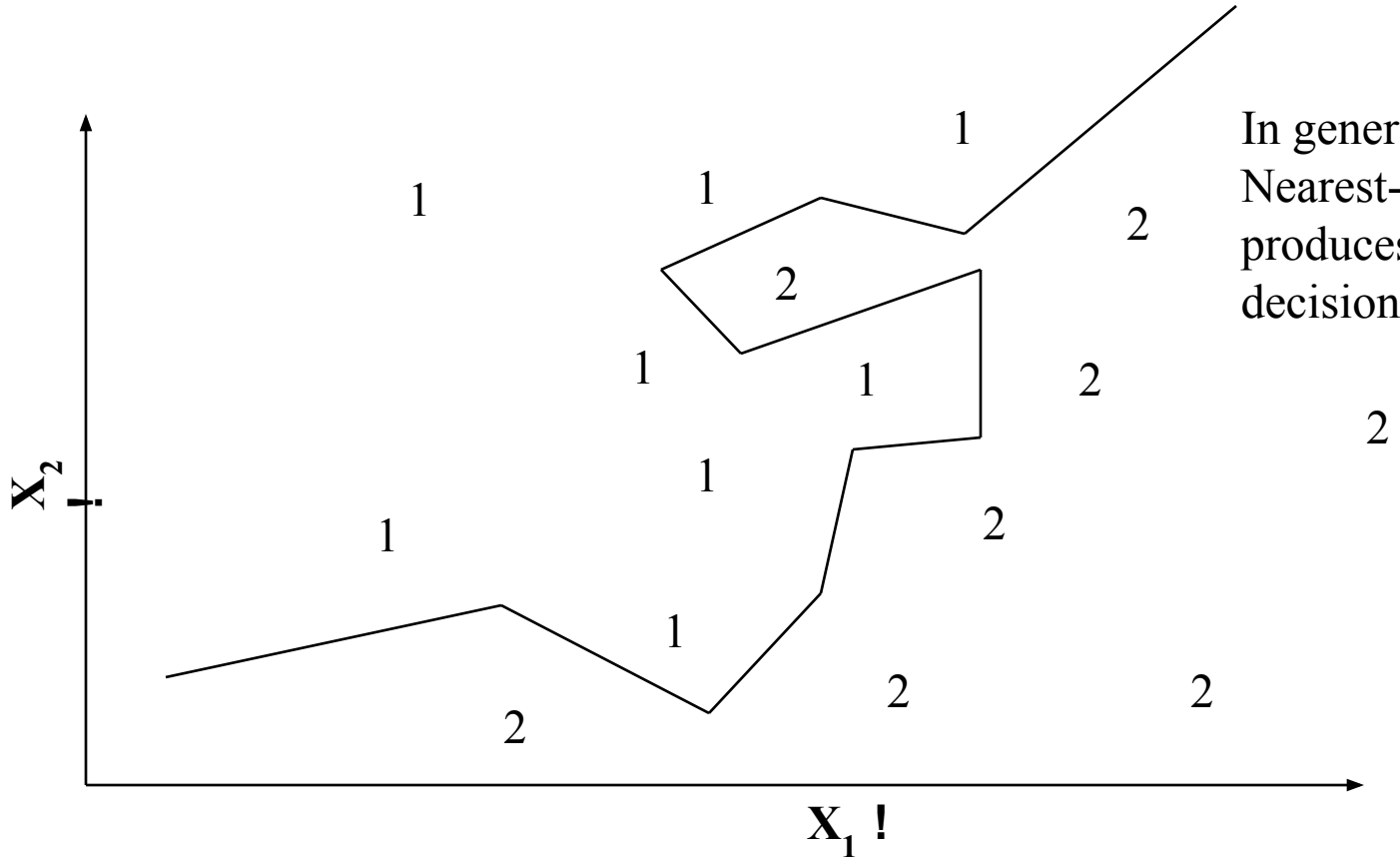
Decision boundary: Those edges across which the decision (class of nearest training datum)

Nearest Nbr:
Piecewise linear boundary

More Data Points

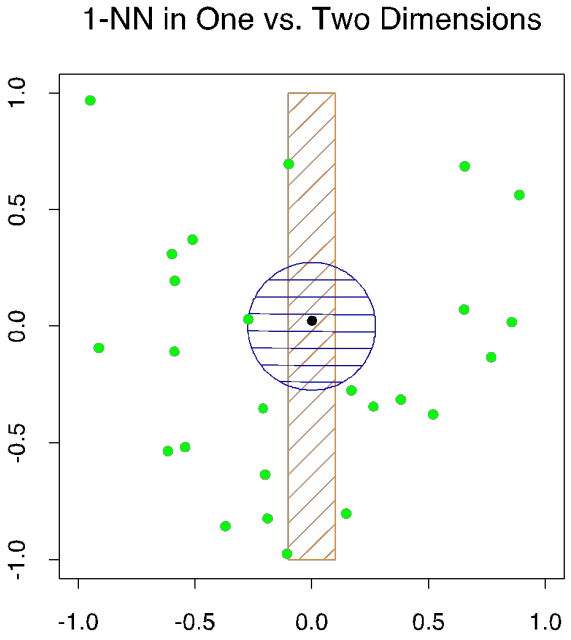


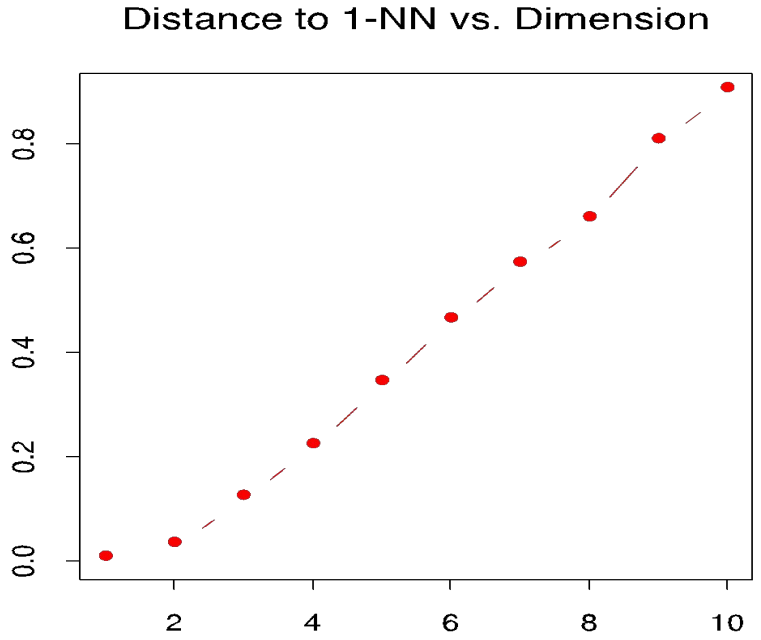
More Complex Decision Boundary



In general:
Nearest-neighbor classifier produces piecewise linear decision boundaries

Issue: Neighbor Distance & Dimension





For m=1000 data in unit hypercube

Machine Learning

Complexity and Overfitting

Nearest Neighbors

K-Nearest Neighbors

K-Nearest Neighbor (kNN) Classifier

Find the k-nearest neighbors to $\underline{x}$ in the data

- i.e., rank the feature vectors according to Euclidean distance
- select the k vectors which are have smallest distance to $\underline{x}$

Regression

- Usually just average the y-values of the k closest training examples

Classification

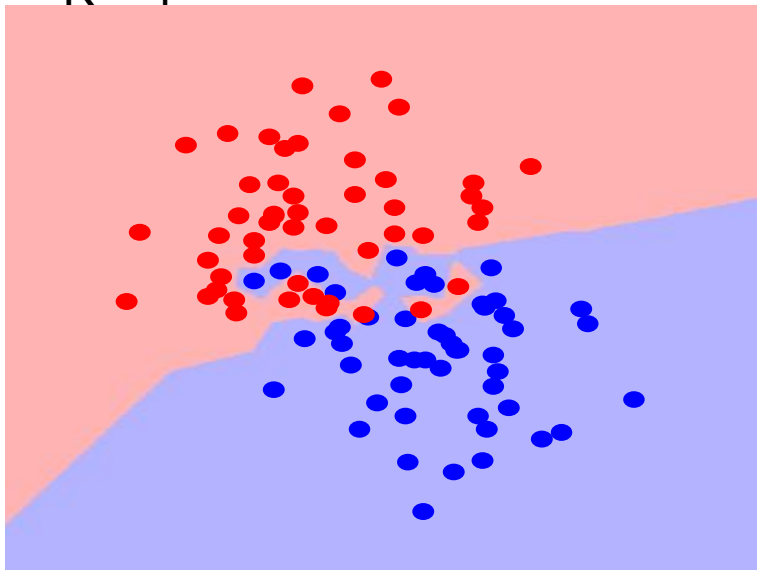
- ranking yields k feature vectors and a set of k class labels
- pick the class label which is most common in this set ("vote")
- classify $\underline{x}$ as belonging to this class
- Note: for two-class problems, if k is odd (k=1, 3, 5, …) there will never be any "ties"; otherwise, just use (any) tie-breaking rule

"Training" is trivial: just use training data as a lookup table, and search to classify a new datum. Can be computationally expensive: must find nearest neighbors within a potentially large training set

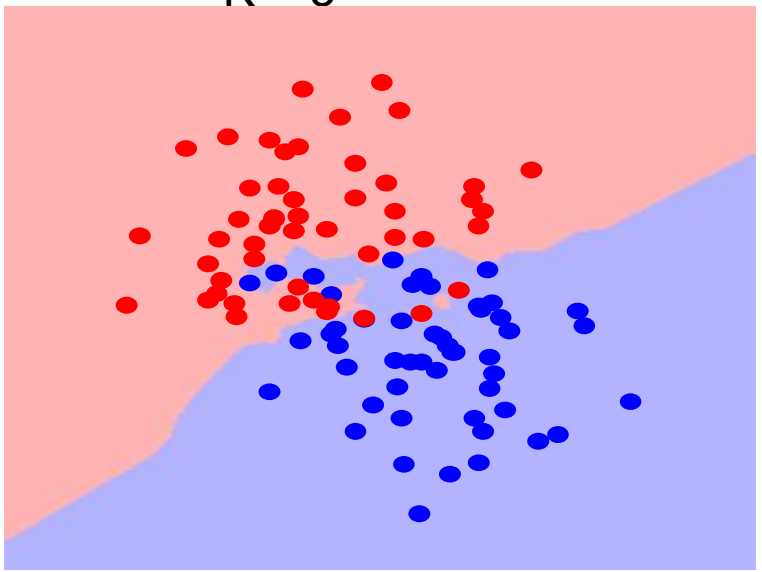
kNN Decision Boundary

- Piecewise linear decision boundary
- Increasing k “simplifies” decision boundary
  - Majority voting means less emphasis on individual points

K = 1



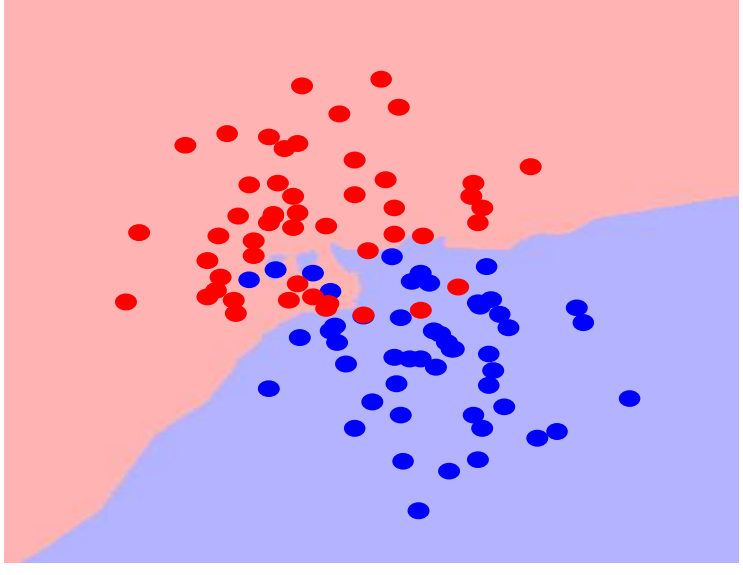
K = 3



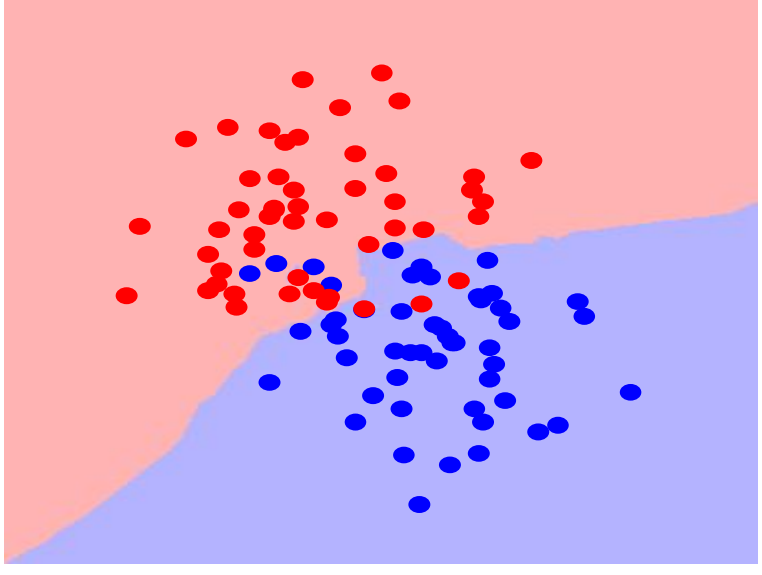
kNN Decision Boundary

- Piecewise linear decision boundary
- Increasing k “simplifies” decision boundary
  - Majority voting means less emphasis on individual points

K = 5



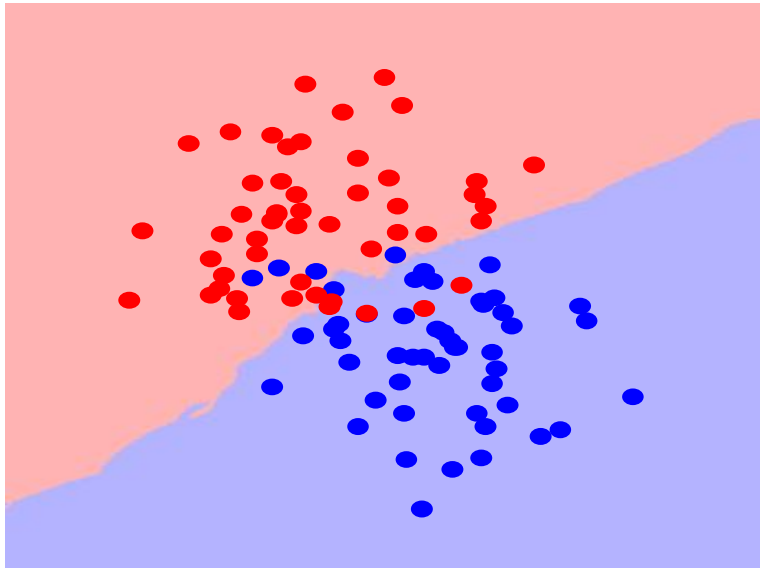
K = 7



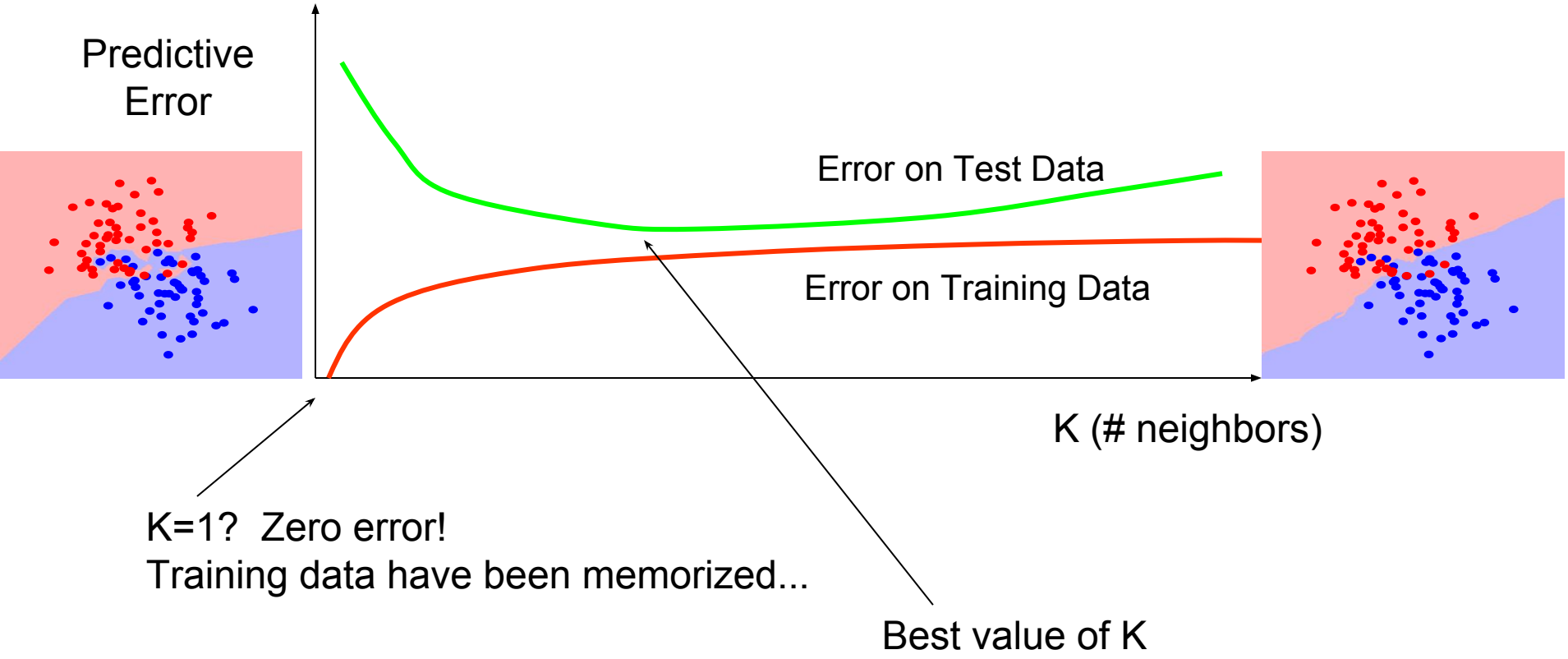
kNN Decision Boundary

- Piecewise linear decision boundary
- Increasing k “simplifies” decision boundary
  - Majority voting means less emphasis on individual points

K = 25

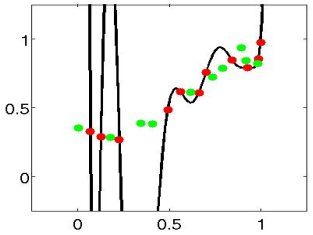


Error rates and K

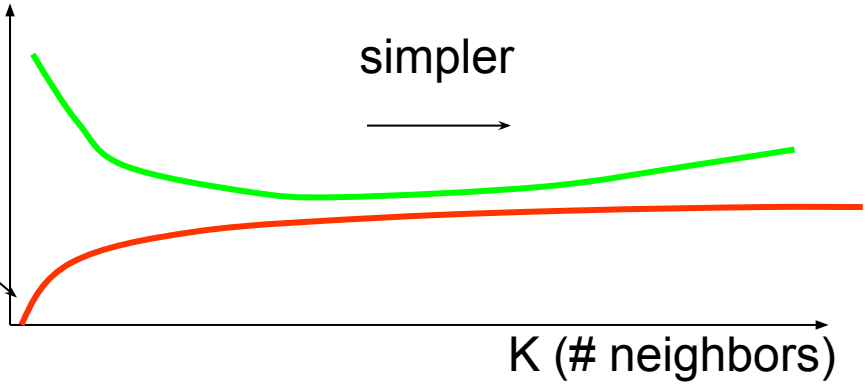


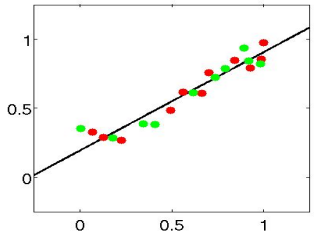
Complexity & Overfitting

- Complex model predicts all training points well
- Doesn't generalize to new data points
- k = 1 : perfect memorization of examples (complex)
- k = m : always predict majority class in dataset (simple)
- Can select k using validation data, etc.



Too complex





K-Nearest Neighbor (kNN) Classifier

Theoretical Considerations

- as k increases
  - we are averaging over more neighbors
  - the effective decision boundary is more “smooth”
- as m increases, the optimal k value tends to increase

Extensions of the Nearest Neighbor classifier

- Weighted distances $d(x, x') = \sqrt{\sum_i w_i (x_i - x'_i)^2}$
  - e.g., some features may be more important; others may be irrelevant
- Fast search techniques (indexing) to find k-nearest points in d-space
- Weighted average / voting based on distance

Summary

- K-nearest neighbor models
  - Classification (vote)
  - Regression (average or weighted average)
- Piecewise linear decision boundary
  - How to calculate
- Test data and overfitting
  - Model “complexity” for knn
  - Use validation data to estimate test error rates & select k

Upcoming

- Sign up on Piazza
  - All course announcements will be done through Piazza! You may miss information announcements if you are not on Piazza.

- Homework 1 will be up soon.
  - Meanwhile, get familiar with Python, Numpy, Matplotlib

Acknowledgement

Based on slides by Alex Ihler & Sameer Singh