

CS142B Language Processor Construction

Building Static Single Assignment Form

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April 16, 2019

Static Single Assignment Form

- A property of an intermediate representation
- Each variable is assigned exactly once
- Control-flow and data-flow are explicit
 - Simplifies and improves a variety of compiler optimizations

A Simple Example

$$y = 1$$

$$y = 2$$

$$x = y$$

A Simple Example

$$y = 1$$

$$y = 2$$

$$x = y$$



$$y_1 = 1$$

$$y_2 = 2$$

$$x_1 = y_2$$

A Simple Example

$$y = 1$$

$$y = 2$$

$$x = y$$



$$y_1 = 1$$

$$y_2 = 2$$

$$x_1 = y_2$$

Statically Unknown Use

```
if(x > 4){  
    y = x - 2;  
} else{  
    y = x + 9;  
}  
w = x + y;
```

```
if(x1 > 4){  
    y1 = x1 - 2;  
} else{  
    y2 = x1 + 9;  
}  
w1 = x1 + y?;
```

Φ Function

```
if(x > 4){  
    y = x - 2;  
} else{  
    y = x + 9;  
}
```

```
w = x + y;
```

```
if(x1 > 4){  
    y1 = x1 - 2;  
} else{  
    y2 = x1 + 9;  
}
```

```
y3 =  $\Phi$ (y1, y2);
```

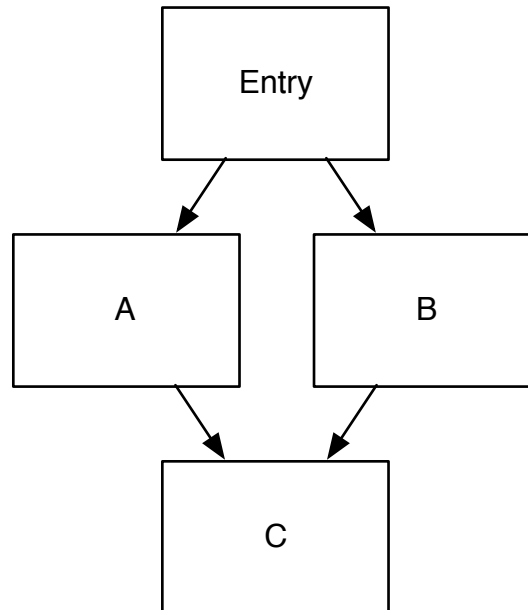
```
w1 = x1 + y3;
```

Where to put these Φ functions?

- Insert Φ for every definition at every join node
- How to find the minimal join nodes
 - Dominance Frontiers

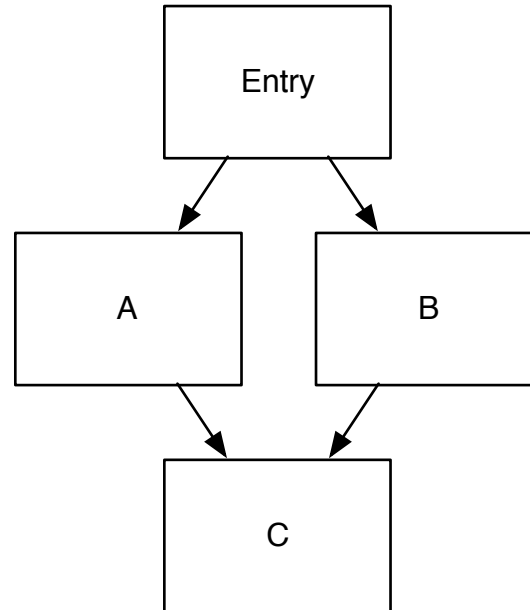
Dominance Property

- Dominators
 - X dominates Y if all paths from *Entry* to Y must pass X
 - X strictly dominates Y , if X dominates Y and $X \neq Y$
- In SSA form, definitions must dominate uses!



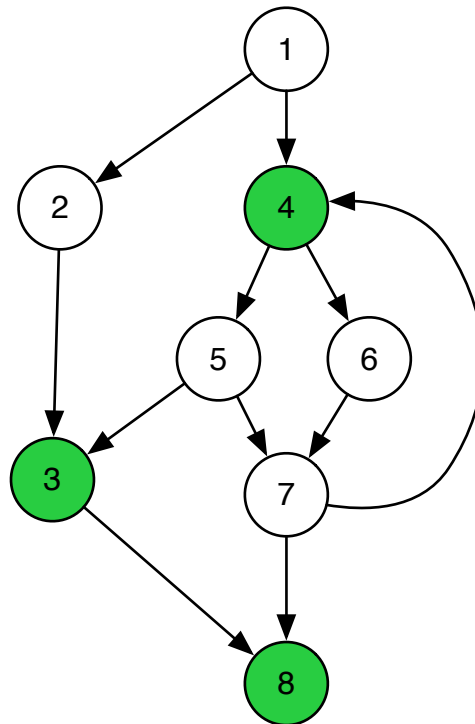
Dominance Frontier

- Y is a dominance frontier of a node X iff
 - X dominates a predecessor of Y , and
 - X does not strictly dominate Y



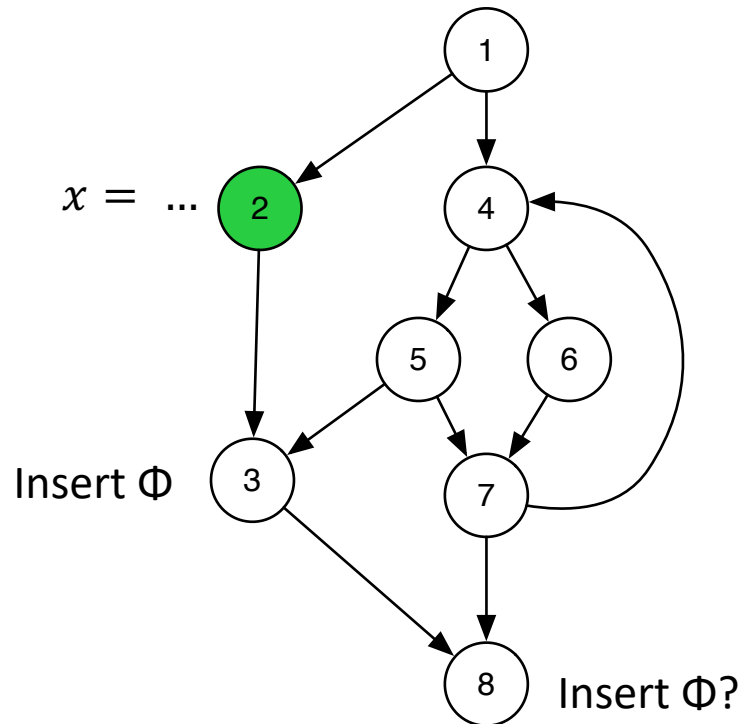
Dominance Frontier Example

- $DF(d) = \{n \mid \exists p \in pred(n), d \text{ dom } p \text{ and } d \neq sdom\ n\}$
- $Dom(4) = \{4, 5, 6, 7\}$
 - $iSucc(4) = \{5, 6\}$, $iSucc(5) = \{3, 7\}$, $iSucc(6) = \{7\}$, $iSucc(7) = \{4, 8\}$
- $DF(4) = \{3, 4, 8\}$



Φ Function Exercise

- $Dom(2) = \{2\}, iSucc(2) = \{3\} = DF(2)$



Algorithm for Inserting Φ Functions

```
for each variable  $v$ 
  WorkList  $\leftarrow \emptyset$ 
  EverOnWorkList  $\leftarrow \emptyset$ 
  AlreadyHasPhiFunc  $\leftarrow \emptyset$ 
  for each node  $n$  containing an assignment to  $v$ 
    WorkList  $\leftarrow$  WorkList  $\cup \{n\}$ 
  EverOnWorkList  $\leftarrow$  WorkList
  while WorkList  $\neq \emptyset$ 
    Remove some node  $n$  from WorkList
    for each  $d \in DF(n)$ 
      if  $d \notin$  AlreadyHasPhiFunc
        Insert a  $\phi$ -function for  $v$  at  $d$ 
        AlreadyHasPhiFunc  $\leftarrow$  AlreadyHasPhiFunc  $\cup \{d\}$ 
      if  $d \notin$  EverOnWorkList
        WorkList  $\leftarrow$  WorkList  $\cup \{d\}$ 
        EverOnWorkList  $\leftarrow$  EverOnWorkList  $\cup \{d\}$ 
```

Algorithm for Inserting Φ Functions

- For each variable,
 - Find all basic blocks that define the variable
 - For each of these basic blocks,
 - Find its dominance frontiers
 - Insert Φ for each of the dominance frontier

Build SSA Step by Step

- Build CFG
 - Create basic blocks
 - Link basic blocks with successors, predecessors
 - Fill the basic blocks with instructions
- Insert Φ nodes
- Rename variables to be in SSA form

Create Basic Blocks

1. Determine a set of leaders, the first instruction of basic blocks
 1. The first instruction is a leader
 2. Instruction L is a leader if it's a target of a branch instruction
 3. Instruction L is a leader if it immediately follows a branch instruction
2. A basic block consists of a leader and all the following instructions until the next leader

Example Basic Block Creation

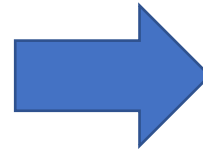
```
0: iconst_0
1: istore_1
2: iload_1
3: bipush          20
5: iadd
6: istore_2
7: iload_2
8: istore_3
9: iload_1
10: ifne           16
13: bipush         53
15: istore_3
16: iload_3
17: invokestatic  #4
20: return
```

Example Basic Block Creation

```
Leader  0: iconst_0
        1: istore_1
        2: iload_1
        3: bipush          20
        5: iadd
        6: istore_2
        7: iload_2
        8: istore_3
        9: iload_1
       10: ifne            16
       13: bipush          53
       15: istore_3
       16: iload_3
       17: invokestatic   #4
       20: return
```

Example Basic Block Creation

```
Leader 0: iconst_0  
1: istore_1  
2: iload_1  
3: bipush          20  
5: iadd  
6: istore_2  
7: iload_2  
8: istore_3  
9: iload_1  
10: ifne           16  
Leader 13: bipush    53  
15: istore_3  
Leader 16: iload_3  
17: invokestatic  #4  
20: return
```



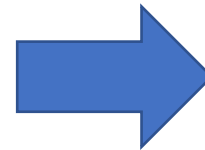
```
BB#0  
StartIndex = 0
```

```
BB#1  
StartIndex = 13
```

```
BB#2  
StartIndex = 16
```

Example Basic Block Creation

Leader	0: iconst_0	
	1: istore_1	
	2: iload_1	
	3: bipush	20
	5: iadd	
	6: istore_2	
	7: iload_2	
	8: istore_3	
	9: iload_1	
	10: ifne	16
Leader	13: bipush	53
	15: istore 3	
Leader	16: iload_3	
	17: invokestatic	#4
	20: return	



BB#0
StartIndex = 0

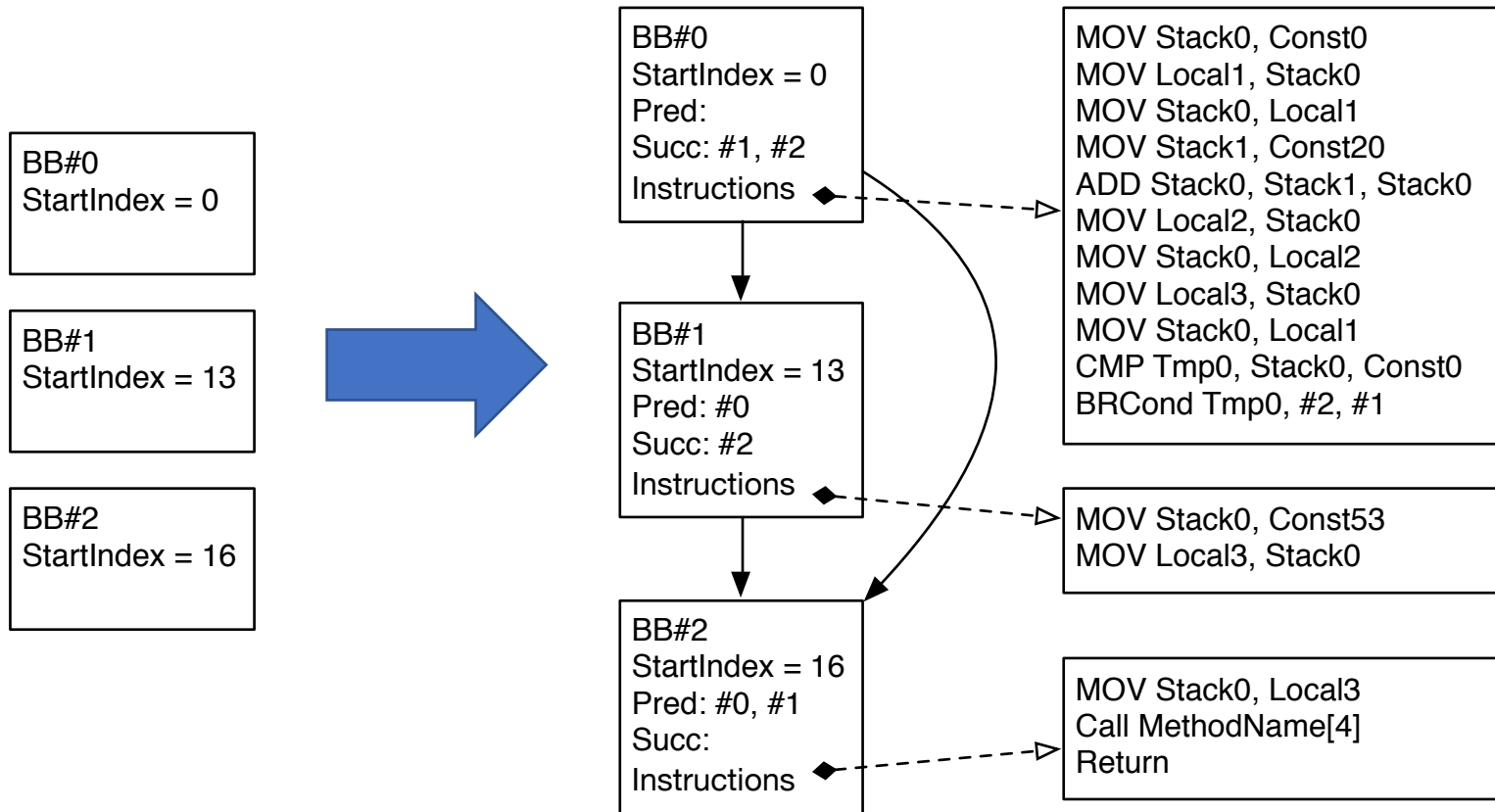
BB#1
StartIndex = 13

BB#2
StartIndex = 16

Link and Fill Basic Blocks

- Each basic block contains a list of successors and a list of predecessors
- At the same time, fill basic blocks with instructions

Link and Fill Basic Blocks



Intermediate Representation

- Two-Address Code
 - E.g., add %0, %1
- Three-Address Code
 - E.g., add %0, %1, %2
 - Quadruples: <Opcode, Op1, Op2, Op3>
- Deal with Stack : make def/use of data on the operand stack explicit
 - Stack Numbering

Intermediate Representation

- Value: empty parent class
- AbstractVariable: inherits from Value
- Instruction (Opcode) inherits from Value
 - The following lists inherit from Instruction
 - MoveInstruction (Opcode, Op1, Op2)
 - BinaryInstruction (Opcode, Op1, Op2, Op3)
 - CmpInstruction (Opcode, Cond, Op1, Op2)
 - CondBranchInstruction (Opcode, Cond, TargetBB1, TargetBB2)
 - UncondBranchInstruction (Opcode, TargetBB)
 - CallInstruction (Opcode, TargetMethod)
 - ReturnInstruction (Opcode, Op1)
 - PhiInstruction (Opcode, {BB1, Op1}, {BB2, Op2}, ...)

Stack Numbering

LiveInStackDepth : 0

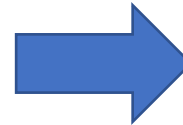
```
0: iconst_0           StackDepth++ : 1
1: istore_1          StackDepth-  : 0
2: iload_1           StackDepth++ : 1
3: bipush            20   StackDepth++ : 2
5: iadd              StackDepth-  : 1
6: istore_2          StackDepth-  : 0
7: iload_2           StackDepth++ : 1
8: istore_3          StackDepth-  : 0
9: iload_1           StackDepth++ : 1
10: ifne             16   StackDepth-  : 0
```

LiveInStackDepth : 0

```
13: bipush           53   StackDepth++ : 1
15: istore_3         StackDepth-  : 0
```

LiveInStackDepth : 0

```
16: iload_3           StackDepth++ : 1
17: invokestatic    #4   StackDepth-  : 0
20: return
```



```
MOV Stack0, Const0
MOV Local1, Stack0
MOV Stack0, Local1
MOV Stack1, Const20
ADD Stack0, Stack1, Stack0
MOV Local2, Stack0
MOV Stack0, Local2
MOV Local3, Stack0
MOV Stack0, Local1
CMP Tmp0, Stack0, Const0
BRCond Tmp0, #2, #1
```

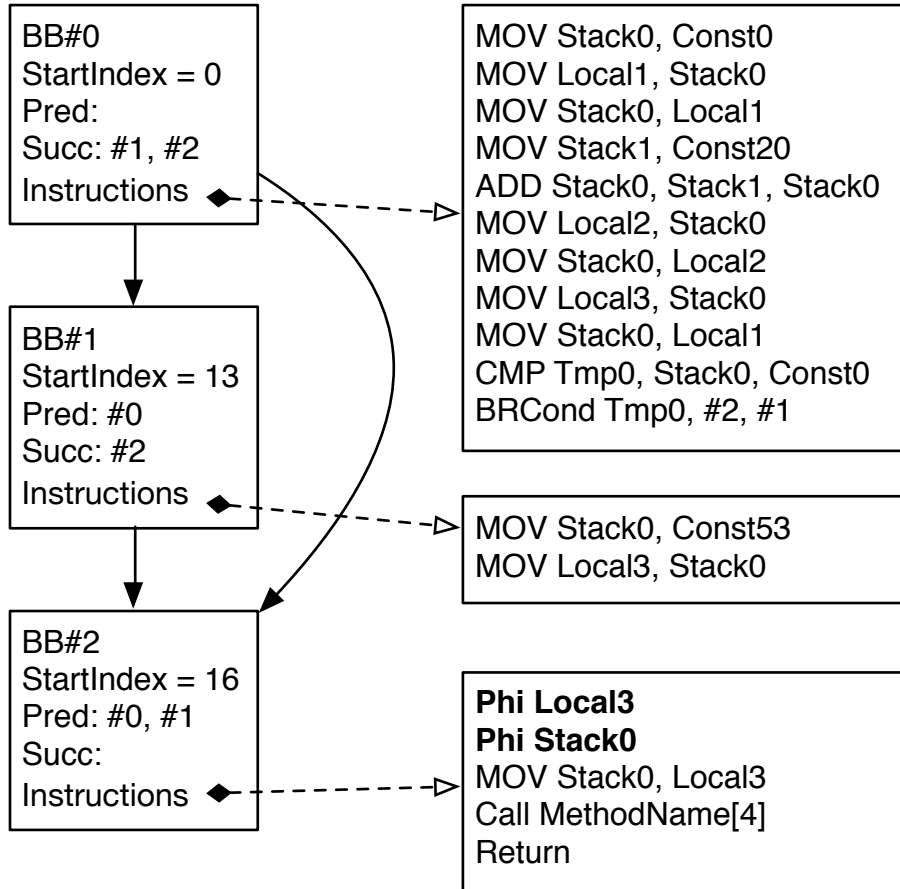
```
MOV Stack0, Const53
MOV Local3, Stack0
```

```
MOV Stack0, Local3
Call MethodName[4]
Return
```

Algorithm for Inserting Φ Functions

- For each variable,
 - Find all basic blocks that define the variable
 - For each of these basic blocks,
 - Find its dominance frontiers
 - Insert Φ for each of the dominance frontier

After Inserting Φ Functions



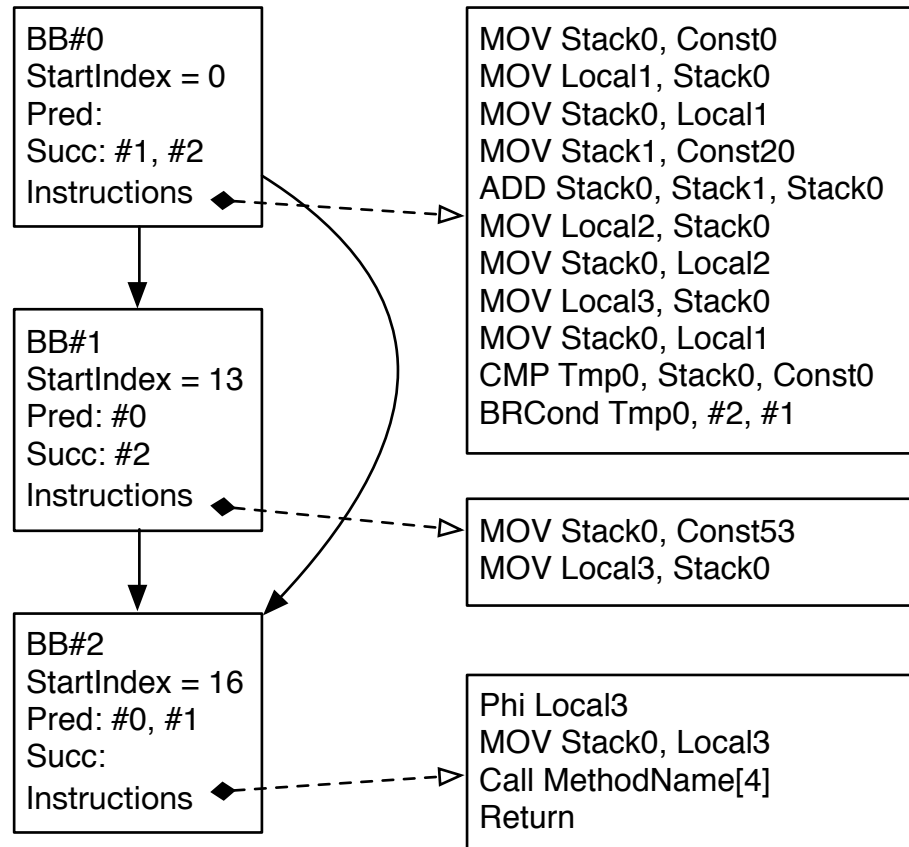
Phi Stack0 is Redundant!

Optimization

- Dead Definition
 - No need to insert Phi, if the definition *LIVE-OUT* for the defining block or *LIVE-IN* for the join block
 - But this requires liveness analysis
- Simple optimization for the operation stack
 - Record the stack depth (live-in stack) at the beginning of each block
 - Do not insert Phi if the live-in stack depth of the dominance frontier is smaller than or equal to the stack variable index

After Phi Optimization

```
0: iconst_0
1: istore_1
2: iload_1
3: bipush      20
5: iadd
6: istore_2
7: iload_2
8: istore_3
9: iload_1
10: ifne      16
13: bipush    53
15: istore_3
16: iload_3
17: invokestatic #4
20: return
```



Variable Renaming (SSA Form)

- For a variable definition, create a new name for it
- For a variable use, use on the top of the name stack
 - i.e., the recent definition in the dominator

$$y = 1$$

$$y = 2$$

$$x = y$$

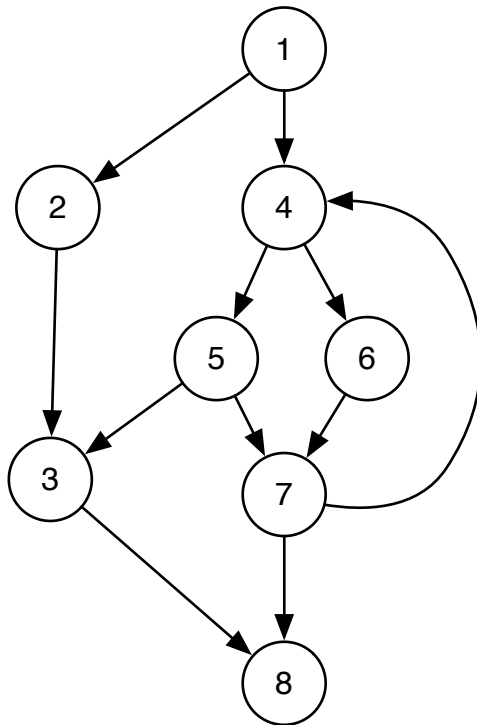
$$y_1 = 1$$

$$y_2 = 2$$

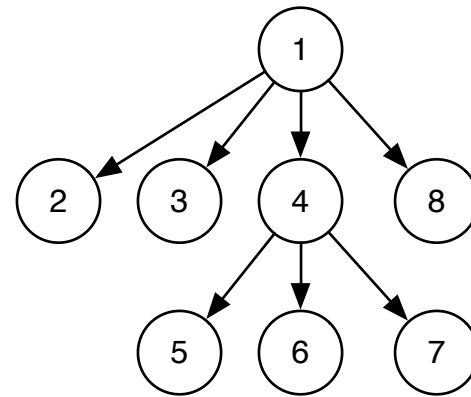
$$x_1 = y_2$$

Dominance Tree

CFG



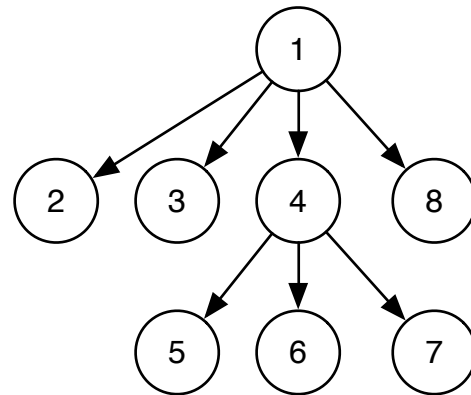
Dominance Tree



Variable Renaming (SSA Form)

- Data Structure
 - $\text{Stacks}[v]$: Holds the most recent definition of variable v , initially empty
- Use Dominance Tree to maintain Variable name stacks

```
procedure GenName(variable v)  
  vn = new Value(v)  
  Push vn onto Stacks[v]  
  return vn
```



Variable Renaming Algorithm

```
call Rename(entry)
```

```
procedure Rename(block b)
```

```
  if b previously visited return
```

```
  for each  $\varphi$ -function p in b
```

```
    v = LHS(p)
```

```
    vn = GenName(v) and replace v with vn
```

```
  for each statement s in b (in order)
```

```
    for each variable v  $\in$  RHS(s)
```

```
      replace v by Top(Stacks[v])
```

```
    for each variable v  $\in$  LHS(s)
```

```
      vn = GenName(v) and replace v with vn
```

```
  for each s  $\in$  succ(b) (in CFG)
```

```
    j  $\leftarrow$  position in s's  $\varphi$ -function corresponding to block b
```

```
  for each  $\varphi$ -function p in s
```

```
    replace the jth operand of RHS(p) by Top(Stacks[v])
```

```
  for each s  $\in$  child(b) (in DT)
```

```
    Rename(s)
```

```
  for each  $\varphi$ -function or statement t in b
```

```
    for each vi  $\in$  LHS(t)
```

```
      Pop(Stacks[vi])
```

Replace with
most recent
definition of v
On the stack

DFS in DT

Unwind the
stack

After Variable Renaming

After Renaming Variables (in two different representations)

```
MOV Stack0_1, Const0
MOV Local1_1, Stack0_1
MOV Stack0_2, Local1_1
MOV Stack1_1, Const20
ADD Stack0_3, Stack1_1, Stack0_2
MOV Local2_1, Stack0_3
MOV Stack0_4, Local2_1
MOV Local3_1, Stack0_4
MOV Stack0_5, Local1_2
CMP Tmp0, Stack0_5, Const0
BRCond Tmp0, BB#2, BB#1
```

```
#1 : MOV Const0
#2 : MOV (#1)
#3 : MOV (#2)
#4 : MOV Const20
#5 : ADD (#4), (#3)
#6 : MOV (#5)
#7 : MOV (#6)
#8 : MOV (#7)
#9 : MOV (#6)
#10 : CMP Tmp0, (#9), Const0
BRCond (#10), BB#2, BB#1
```

```
MOV Stack0_6, Const53
MOV Local3_2, Stack0_6
```

```
#12 : MOV Const53
#13 : MOV (#12)
```

```
Phi Local 3_3, (Local3_1, BB#1), (Local3_2, BB#2)
MOV Stack0_7, Local3_3
Call MethodName[4]
Return
```

```
#14 : Phi ((#8), BB#1), ((#13), BB#2)
#15 : MOV (#14)
Call MethodName[4]
Return
```

Quadruples

Triples

Implementation Example

```
class Value {  
    bool isInstruction;  
    ...  
};
```

```
class AbstractVariable : public Value {  
    bool isStack;  
    int Index;  
    ...  
};
```

```
class Instruction : public Value {  
    Opc;  
    bool hasLHS;  
};
```

```
class BinaryInstruction : public Instruction {  
    Value *Op1; Value *Op2; Value *Op3;  
};
```

```
class PhiInstruction : public Instruction {  
    Value *Dst;  
    pair<BasicBlock*,Value*> InEdges[]  
    ...  
};  
...
```

Summary

- SSA form - explicit use-def relationships
- Build CFG
 - Create Basic Blocks with the identified leaders
 - Link Basic Blocks and fill them with instructions
 - Rename operand stack variables
 - Instructions are not in SSA form yet!
- Insert Phi Functions based on the dominance frontier algorithm
- Rename variables to have SSA property

Interpreting with SSA IR for fun