STAT120A Assignment 2

Instructions: Write your answers on letter-sized sheets. Your work should be well-organized and easy to read. You must show your work, rather than only provide the final answer, to receive full credit.

Part I:

1. A coin is tossed three times and the sequence of heads and tails is recorded.
   (a) List the sample space Ω.
   (b) List the elements/outcomes that make up the following events
      i. $A = \text{at least two heads.}$
      ii. $B = \text{the first two tosses are heads.}$
      iii. $C = \text{the last toss is a tail.}$
   (c) List the elements/outcomes of the following events: $A^c, A \cap B, A \cup C$

2. Two six-sided dice are thrown sequentially, and the face value that come up are recorded.
   (a) List the sample space Ω.
   (b) List the elements that make up the following events:
      i. $A = \text{the sum of the two values is at least 5.}$
      ii. $B = \text{the value of the first die is higher than the value of the second.}$
      iii. $C = \text{the first value is 4.}$
   (c) List the elements of the following events : $A \cap C, B \cup C, A \cap (B \cup C)$.

3. Prove the Bonferroni’s inequality:
   $$P(A \cap B) \geq P(A) + P(B) - 1$$

4. Let $A$ and $B$ be events with probabilities $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$. Show that $\frac{1}{12} \leq P(A \cap B) \leq \frac{1}{3}$.

5. The weather forecaster says that the probability of rain on Saturday is 25% and that the probability of rain on Sunday is 25%. Is the probability of rain during the weekend 50%? Why or why not?

6. What is the coefficient of $x^3y^4$ in the expansion of $(x + y)^7$?

7. A child has seven blocks, three of which are red and four of which are green. How many patterns can she make by placing them all in a line?

Part II: The sample function in R takes a sample of a specified size from the elements given in its first argument. Answer the following questions:

1. Look up the help file for the sample function. What does the argument “replace” do in the function?

2. Use the sample function to simulate five rolls of a six-sided (fair) dice. To ensure reproducibility during grading, please run `set.seed(your ID)` right before you run the sample function. Here you ID is the number of your student ID. For example, if your student ID is ”123456”, you should run `set.seed(123456)` in R.

3. Use the sample function to simulate 1,000 rolls of a six-sided (fair) dice. Using your results to count the frequencies of the six numbers. Again, to ensure reproducibility, run `set.seed(your ID)` right before you run the sample function.

1